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Genetics. — J. F. VAN BEMMELEN: *Geslachtelijke en Ongeslachtelijke Voortplanting.*

(Communicated at the meeting of December 21, 1929).

In vrijwel alle handboeken der ontwikkelingsgeschiedenis begint het hoofdstuk over de Voortplanting met de opmerking, dat men daarvan twee soorten kan onderscheiden: de geslachtelijke en de ongeslachtelijke. Gewoonlijk wordt aan de eerstgenoemde verreweg de meeste aandacht geschonken, terwijl de ongeslachtelijke voortplanting pas in de tweede plaats en veel minder uitvoerig wordt behandeld, waardoor, zij het onwillekeurig, de indruk wordt gewekt, alsof deze laatste wijze van vermenigvuldiging niet slechts minder algemeen zou voorkomen dan de geslachtelijke, maar ook tot zekere hoogte als een abnormaal proces ware op te vatten, dat slechts onder bijzondere omstandigheden en eenigermate bij uitzondering optrad. Deze omstandigheden zouden dan nog vaak voor het welzijn en het voortbestaan van de soort van ongunstigen aard blijken.

Tegen deze voorstelling nu wensch ik krachtig op te komen. Ik meen dit het best te kunnen doen, door mijne inzichten in eenige stellingen zoo scherp en beslist mogelijk tot uitdrukking te brengen, op gevaar af daardoor aan mijn beweringen een ietwat apodictisch en absoluut, ja zelfs eenigszins paradox karakter te geven.

Voortplanting of vermenigvuldiging is een levensverschijnsel, dat zich bij alle organismen, planten zoowel als dieren, op een en dezelfde wijze voordoet, n.l. verdeling in stukken. Het aantal dezer stukken kan zoowel twee als meer bedragen. Elk deelstuk vervormt zich na zijn afscheiding op de een of andere wijze tot een volledig organisme, dat in de meerderheid der gevallen overeenkomt met het ouder-organisme waaruit het ontstaan is.

Bepalen wij voor de eenvoudigheid onze aandacht eerst uitsluitend bij de tweedeling, dan kunnen we beweren, dat de twee stukken waarin een organisme zich splitst, nimmer volkomen aan elkaar gelijk zijn, maar altijd van elkaar verschillen, en dat dit onderscheid met meer of minder noodzakelijkheid voortvloeit uit den polairen, bilateraal symmetrischen bouw, die aan alle levende wezens zonder onderscheid eigen is, ook aan diegene, welke oogenschijnlijk volgens een anderen symmetrie-regel, b.v. den straalsgewijzen, gebouwd zijn, of waarbij zelfs alle symmetrie schijnt te ontbreken, zooals bij de meeste Amoeben. Bijzonder duidelijk laat zich dit onderscheid aantoonen bij de Infusoriën, b.v. *Paramecium*, waarbij de deeling in tweeën plaats vindt volgens een dwarsvlak loodrecht op de lange as, en het diertje zich dus splitst in een vóór- en een achterstuk. Na het loslaten der beide stukken van elkaar moet dus de vóórheft een nieuw achter-, de achterheft een nieuw voorstuk regenereren.

Niettegenstaande dit onderscheid geeft de aard dezer deeling geen aanleiding tot het opkomen der voorstelling van ouder en kind. Dat is dan ook de reden, waardoor AUGUST WEISMANN gekomen is tot zijn opvatting, als zouden de Eencellige wezens potentieel onsterfelijk zijn, omdat bij hen de seniele involutie zou ontbreken, aangezien zij zich bij iedere deeling verjongden. Het verband met de meercellige organismen, bij welke deze ouderdomsverwording naar zijn meening nimmer ontbrak, meende WEISMANN te kunnen verklaren, door bij hen onderscheid te maken tusschen somatisch en kiemplasma, en alleen aan dit laatste de potentieele onsterfelijkheid toe te kennen, terwijl het somatisch plasma noodwendigerwijze na korteren of langeren tijd, tengevolge van constitutioneele veranderingen, te gronde ging.

Naar mijne opvatting bestaat er geen reden om een dergelijk principieel onderscheid tusschen één- en meercellige organismen aan te nemen. In de eerste plaats niet, omdat de vraag of een levend wezen uit één of uit meer cellen is opgebouwd, van ondergeschikte beteekenis is bij de beoordeeling van den eigenlijken aard zijner organisatie. Deze opvatting, waarbij alzoo de tegenstelling tusschen Protozoen en Metazoen (resp. Protophyten en Metaphyten) komt te vervallen, werd door mij toegelicht in mijn voordracht: *Organismenbegriff und Zellenlehre*, op het Internationale Zoölogen-Congres te Boedapest in 1927, en mag ik dus hier laten rusten.

In de tweede plaats echter, omdat er geen reden is een onverbreekbaren samenhang aan te nemen tusschen het verschijnsel van den dood en de seniele involutie. Mijns inziens is de dood een primair levensverschijnsel, evengoed als de overige karakteristika van het Leven: stofwisseling, groei, vervorming, voortplanting.

Men kan dit ook zoo uitdrukken: niet de wijze, waarop een organisme afsterft, maar het feit dat het kan sterven op zichzelf, is het belangwekkende. Deze eigenschap nu: de vatbaarheid om onder bepaalde omstandigheden over te gaan tot de levenlooze wereld, met welken overgang aan het bestaan als zelfstandig organisme voorgoed een eind komt, is inhaerent aan alle levende wezens zonder onderscheid. Welke die omstandigheden zijn, is een zaak van ondergeschikte beteekenis: altijd en overal zijn zij zoowel afhankelijk van uitwendige als van inwendige invloeden en gesteldheid, maar nimmer uitsluitend van een van beide. Daaruit volgt, dat het eigenlijk niet aangaat, om onderscheid te maken tusschen natuurlijke en kunstmatigen dood, en zeker niet om te meenen, dat de eerstgenoemde bij de Protozoen en Protophyten zou ontbreken.

Terugkeerend tot de twee-deeling, zij opgemerkt, dat aan de gevallen, waarin de deelstukken in vorm en omvang niet noemenswaard van elkaar verschillen, en er dus geen sprake van is, dat het ééne stuk zich uit het andere zou ontwikkeld hebben, zich andere aansluiten, waarbij dit laatste wel degelijk het geval is. Bijzonder duidelijke voorbeelden ervan ontmoeten wij bij de Thecamoeben, o.a. in het door SCHEWIAKOFF onderzochte geval van *Euglypha*, bij welke uit de monding van de uit losse kiezelstukjes

bestaande pantserbekleding een uitpuiling van reticulair plasma uitgroeit. Deze bereikt den omvang en den vorm van het oorspronkelijke organisme, en vormt dus daarvan het spiegelbeeld, terwijl zij er zoolang mee blijft samenhangen, totdat hare organisatie aan die van haar voortbrengster gelijk is geworden. Al wat zij daartoe behoeft, wordt haar door deze laatste toegevoerd: eerst pantserplaatjes, die in het plasma van de voortbrengster worden gevormd, en door plasmastroomen naar den knop overgebracht; vervolgens voedseldeeltjes, en ten slotte een kern, die zich door karyokinese uit de oorspronkelijke heeft ontwikkeld. Daarna laten de beide individuen van elkaar los, onder een levendig spel van pseudopodien-uitstulping.

Reeds het feit, dat wij hier van een „oorspronkelijke” kern, en van een „voortbrengster” mogen spreken, bewijst dat de onderscheiding van ouder en kind zich in dit geval om zoo te zeggen vanzelf aan ons opdringt.

In nog hoogere mate is dit het geval bij de sporevorming der Protisten, waarbij een ééncellig organisme uiteenvalt in meer dan twee deelstukken, welk aantal tot in de duizenden kan stijgen. Met deze sterkere vermenigvuldiging gaat in de meeste gevallen een aanzienlijker onderscheid tusschen het sporulerende organisme en de sporen, in welke het uiteenvalt, gepaard, waardoor nog meer aanleiding wordt gegeven tot de opvatting, dat de sporen de jongen zijn van het hen voortbrengende wezen. Bovendien blijft van dit laatste dikwijls een gedeelte over, dat niet aan de sporevorming deelneemt, en na afloop van dit proces in ontbinding overgaat. Blijkbaar is het dus niet meer levensvatbaar, wat ons het recht geeft te spreken van afsterving tengevolge van inwendige omstandigheden, dus van een z.g. natuurlijke dood, onder onmiskenbare vorming van een lijk.

Overgaande tot de wereld der Metazoën, mogen wij beweren, dat daarin al de bovenaangeduide verschijnselen hunne volkomen analoga vinden, en hetzelfde geldt voor de Metaphyten. Beperken wij ons ook hier weer tot de tweedeeling, en kiezen wij uit den rijkdom van gevallen enkele der allertreffendste voorbeelden, dan kunnen wij b.v. wijzen op de door-middensnoering der zee-anemonen, waarbij tusschen de beide deelstukken, hoezeer ook overeenkomend in vorm, grootte en toestand van volwassenheid, toch van den aanvang af duidelijke en principieele verschillen bestaan, omdat de insnoering plaats vindt in een vlak loodrecht op de mondsleuf, waardoor aan de eene deelhelft de instroomings-, aan de tweede de uitstroomings-sipho ten deel valt, en elk dus de andere trilhaarsleuf moet regenereren.

Ook dwarsdeeling komt voor, waarbij het bovenstuk loslaat van het onderstuk, welk laatste een nieuwe tentakelkrans doet ontspruiten rondom een nieuwe mondopening, van welke een nieuw stomodaeum naar binnen groeit. Hier is dus het oorspronkelijk onderscheid tusschen de beide deelstukken nog veel opvallender.

Even sterk springt dit onderscheid in het oog bij de vermenigvuldiging door deeling bij Ringwormen. Bij vele marine Polychaeten snoert zich het voorste deel van het veelledige wormlichaam af van het achterste. Aan

het eerste ontstaat op de plaats van afsnoering een nieuw achtereind met anus, aan het laatste een nieuwe kop, en deze regeneratie kan zoowel plaatsvinden vóór als na het loslaten van de beide helften. Daarbij verschilt van den aanvang af de voor- in meerdere opzichten van de achterhelft, bepaaldelijk in de aanwezigheid van geslachtsorganen in de laatstgenoemde, terwijl deze in de voorhelft ontbreken. Het achtereind van de voorhelft gaat nu nieuwe segmenten vormen, en na eenigen tijd snoeren deze zich op hun beurt van het voorstuk af. Deze nieuwvorming kan ook geschieden terwijl het eerstgevormde achterstuk nog met het voorstuk samenhangt, en zoo kan een keten van zes of zelfs meer wormindividueen ontstaan.

Hetzelfde verschijnsel doet zich voor in een geheel andere groep van wormachtige dieren, n.l. bij de Microstomiden onder de Turbellarii. Daar-aan laat zich dan de proglottidenvorming der Cestoden weer aansluiten, ofschoon hier, waarschijnlijk onder den invloed van de parasitische levenswijze, de afzonderlijke leden het niet tot volledige zelfstandig wordende organismen brengen.

Vergelijkt men nu dit verschijnsel der tweedeeling bij volwassen Metazoen, behorende tot geheel verschillende diergroepen, met de vermenigvuldigingswijze der Infusoriën, dan blijkt het daarmee in alle belangrijke opzichten overeen te komen. Ook bij de zich middendoor deelende Zee-anemonen, Anneliden en Turbellariën is de eene helft van den aanvang af verschillend van de andere, zonder dat daarin een voldoende criterium te vinden ware, om de beide deelstukken als ouder en kind tegenover elkaar te stellen. Acht men dus de Infusoriën door hun tweedeeling verjongd, en in verband daarmee potentieel onsterfelijk, dan zou men het ook deze Metazoen moeten doen.

Ook voor de bovenvermelde vermenigvuldigingswijze der Thecamoeben laat zich een geheel analoog verschijnsel bij vele Metazoen terugvinden, n.l. de voortplanting door middel van knoppen. Uit de talloze verschillende gevallen zij hier enkel de knopvorming bij Tunicaten vermeld, om tweeërlei redenen. In de eerste plaats omdat men gerechtigd is de eigenaardige organisatie dezer dieren voor een groot deel in verband te brengen met de aanwezigheid eener door de huid voortgebrachte omhulling: den mantel. In de tweede plaats omdat de Manteldieren ongetwijfeld naverwant zijn met de Gewervelde Dieren, bij welke, oogenschijnlijk tenminste, de vermenigvuldiging door deeling van het organisme zou ontbreken. Dat er geen reden is aan deze afwezigheid (die zich trouwens evenzeer voordoet bij de Mollusca, de Brachiopoda, en nog verschillende andere afdeelingen) eenige belangrijke beteekenis toe te kennen, blijkt o.a. uit het feit dat ook onder de Tunicaten, bij enkele onderafdeelingen, de vermenigvuldiging door knopvorming niet is aangetroffen, o.a. bij de Appendicularia en de Monascidia.

Gekarakteriseerd wordt de knopvorming der Tunicaten door de ontwikkeling eener uitgroeiing op een bepaald punt van het organisme: de kiemstok of stolo. Aan deze ontstaan nieuwe individuen, wier organisatie

plaats vindt onder den regelenden invloed van die organen van het oorspronkelijke individu, welke zich tot in den kiemstok uitstrekken. In sommige gevallen laten de nieuwe individuen van den kiemstok los en gaan een zelfstandig bestaan voeren, in andere blijven zij er mee samenhangen, zoodat een ketenvormige kolonie ontstaat.

In de bovenvermelde gevallen van vermenigvuldiging door deeling hadden wij steeds te doen met voortplantingsverschijnselen bij volwassen organismen. Daarnaast echter vermag men in tal van diergroepen voorbeelden te vinden van vermeerdering van het individuën-tal door afscheiding van nieuwe kiemen gedurende den loop der ontwikkeling. 't Best bekend is dit verschijnsel bij de Trematoden, waar uit de kiemballen der tot Sporocyst vervormde Myracidium-larve een nieuwe generatie van Rediën ontstaat, en in elk dezer wormpjes op dezelfde wijze een aantal Cercariën zich ontwikkelt. Maar onder hetzelfde gezichtspunt kan men verschijnselen brengen, die zich voordoen aan veel jongere stadiën van ontwikkeling, n.l. aan zich klievende eieren. Het eerst zijn deze verschijnselen opgemerkt door de ontwikkelings-mechanici, toen deze er in slaagden de deelstukken, waarin het bevruchte ei zich bij den aanvang zijner ontwikkeling kliefde, van elkaar los te maken zonder ze dermate te beschadigen, dat zij niet verder levensvatbaar waren. Bij verschillende diersoorten, o.a. Amphioxus en bij kikvorschsoorten, bleken de van elkaar geïsoleerde klievingsbollen met de deeling door te gaan, en elk voor zich tot een volledig larfje te worden, dat echter slechts de helft of zelfs maar een kwart of een achtste der grootte van de normale larve bereikte, al naarmate men de klievingsbollen na de eerste, de tweede of de derde deeling van elkaar had losgemaakt.

Wat aanvankelijk slechts door het experiment scheen te verkrijgen, en dus den indruk van een abnormaal gebeuren maakte, bleek later even goed buiten toedoen van menschelijk ingrijpen, onder normale omstandigheden in den loop der ontwikkeling te kunnen optreden, en voor sommige diervormen, uit de meest verschillende groepen, de gewone, regelmatige wijze van vermenigvuldiging te zijn. Bijzonder opmerkelijk lijkt mij in dit opzicht de voortplanting van sommige soorten der Gordeldieren, bij welke uit een enkel ei een viertal embryonen ontstaat, alle van dezelfde sexe, en binnen een gemeenschappelijk chorion besloten. De verdeeling van het oorspronkelijk enkelvoudige ei in vier onderdeelen blijkt hier niet onmiddellijk na de eerste twee klievingsphasen plaats te vinden, maar treedt eerst op wanneer op de volledig gekliefde kiemschijf de sporen van den embryonaal-aanleg zich beginnen af te teekenen. Dan ontstaat n.l. niet één primitiefstreep, maar twee, diametraal tegenover elkaar, en iets later vertoont zich naast elk dezer weder een embryonaalaanleg, zoodat er in 't geheel twee paren zijn, die langzamerhand door ongelijken interstitieelen groei, op onderling gelijke afstanden, dus in de vier kwadranten van het ei, komen te liggen.

Wat voor de Gordeldieren regel is, komt blijkbaar bij andere Zoogdieren als uitzondering voor, getuige de ééneïge tweelingen. Maar als wij de

handboeken der Teratologie opslaan, kunnen wij ons overtuigen, dat ook bij vogels, reptielen, amphibiëen en visschen gevallen bekend zijn, waarin op één kiemschijf twee, of zelfs meer afzonderlijke embryonen werden aangetroffen. En gaan wij naar de Insekten, dan vinden we daar nog veel treffender voorbeelden: immers, er zijn Sluipwespen bekend, bij welke uit één ei (regelmatig, dus blijkbaar normaal) tot duizend en meer larven ontstaan, die ook hier allen dezelfde sexe bezitten, dus of allen zich tot mannetjes, of allen tot wijfjes ontwikkelen.

Deze waarnemingen geven aanleiding, ons af te vragen of niet het geheele verschijnsel der eiklieving moet opgevat worden als een praematuur vermenigvuldigingsproces, waarbij evenwel de door deeling ontstane nakomelingen met elkander in samenhang blijven, en zodoende ten slotte tot onderdeelen van één enkel organisme worden. In de hoogste mate waarschijnlijk lijkt mij in allen gevallen deze opvatting voor de eigenlijke voortplantingscellen: de oereieren en oer-spermatocyten. Van het eerste oogenblik hunner vorming af zijn zij zelfstandige afzonderlijke organismen, die echter nog gedurende korteren of langeren tijd in samenhang blijven met het andere (meestal grootere en meer gedifferentieerde) deelstuk, waaruit het z.g. soma ontstaat. Zoolang die samenhang blijft bestaan, zijn zij voor hun stofwisseling en voor de bescherming tegen uitwendige invloeden, van dit soma afhankelijk, en doen zich dus voor als organen daarvan (eierstokken en zaadklieren), maar in werkelijkheid staan zij tot het soma in dezelfde verhouding als het achterdeel van een zich door midden snoerend infusorium tot zijn voorstuk.

Bij de meeste dieren worden de oergeslachtscellen reeds in een vroegtijdig stadium der ontwikkeling afgescheiden; bij Zoogdieren derhalve lang voor de geboorte, zoodat men beweren kan dat b.v. de mensch zich reeds voortgeplant heeft nog voordat hij het levenslicht heeft aanschouwd. Het optreden der geslachtsrijpheid is dus niet anders dan het onafhankelijk worden en loslaten der geslachtscellen van hun somatische wederhelft.

Deze gedachtengang voert ons op het gebied der sexueele voortplanting, en doet ons dit betreden met voorstellingen, die geheel in overeenstemming blijken met de hedendaagsche opvatting der beide soorten van geslachtscellen. De moderne biologie toch ziet in eieren en spermatozoiden gelijkwaardige en volledige organismen, elk voor zich geschikt om tot een volwassen individu uit te groeien, maar daarin verhinderd door remmende invloeden, die in de meerderheid der gevallen slechts opgeheven kunnen worden door versmelting der beide geslachtscellen tot een dubbel-individu. Op zichzelf beschouwd is die versmelting juist het omgekeerde van vermenigvuldiging: immers zij vermindert het aantal zelfstandige individuen eener soort met de helft. Dat zij geen essentieel element van het voortplantingsproces is, wordt ons bewezen door de Parthenogenesis. Zodoende komen wij ertoe de sexueele voortplanting te beschouwen als een combinatie van twee tegengestelde levensverschijnselen: de eigenlijke vermenigvuldiging, bestaande in de verdeling van een organisme in twee

of meer stukken, en de bevruchting, welke gekenmerkt wordt door de versmelting van twee onafhankelijke individuen eener zelfde soort met elkaar tot een dubbelwezen. Deze twee in elkaar opgaande individuen kunnen evengoed in vorm, grootte en geaardheid volkomen overeenstemmen, als zoo verschillend mogelijk zijn. De verschillen laten zich volledig terugbrengen op aanpassing aan de uitwendige levensomstandigheden, welke den eisch stellen dat één der conjugeerende individuen klein, slank en bewegelijk zij, teneinde het in staat te stellen het andere, grootere en tragere, op te zoeken en binnen te dringen.

Wanneer deze opvatting van het bevruchttingsproces als een zelfstandige levensuiting der organismen, onafhankelijk van hun vermenigvuldigingsdrang, de juiste is, dan mogen wij verwachten, dat wij hetzelfde verschijnsel: versmelting van twee (of zelfs meer) individuen eener zelfde soort tot één enkel samengesteld wezen, ook buiten samenhang met het voortplantingsproces zullen aantreffen. Op het eerste gezicht echter schijnt dit niet het geval te zijn: altijd volgt op de syngamie (zooals de vereeniging der geslachtscellen tegenwoordig genoemd wordt) een levendig deelingsproces, dat bij de ééncelligen tot een versterkte vermeerdering van individuen leidt, bij de meercelligen daarentegen tot de ontwikkeling van één enkel volwassen en geslachtsrijp wordend wezen. Maar juist dit laatste verschijnsel schenkt m.i. een duidelijk inzicht in het ware karakter der individuenversmelting. Immers de klievingsballen van het zich deelende ei zijn, zooals wij daareven zagen, oorspronkelijk afzonderlijke individuen, die echter gewoonlijk niet zelfstandig worden, maar met elkaar blijven samenhangen. De meerderheid dezer deelstukken, n.l. de soma-cellen, treden zelfs in steeds nauwere betrekking tot elkaar, zoodat zij al spoedig hunne zelfstandigheid grootendeels of geheel en al verliezen en tot een meercellig wezen versmelten. Alleen de voortplantingscellen handhaven van den aanvang af hun zelfstandigheid, en maken zich ten slotte van hun zuster-cellen (het soma) los.

Zoo komen wij tot dezelfde conclusie als vele beoefenaars der erfelijkheidsleer: het kind is de broeder (of zuster) zijner moeder.

Maar ook buiten het gebied der voortplanting laten zich wel enkele verschijnselen opsporen, waarbij versmelting van oorspronkelijk zelfstandige organismen een rol speelt. Zoo b.v. de tijdelijke samenvloeiing van een aantal Sporozoen tot een meerkernig complex, ten einde tezamen een buit, die voor elk hunner te omvangrijk zou zijn, te kunnen overmeesteren en verteren; een verschijnsel waarvoor de Duitschers den naam *Fressgesellschaft* hebben uitgedacht.

Zoolang de deelstukken, waarin zich een organisme splitst, nog niet geheel en al van elkaar zijn losgeraakt, blijven zij invloed op elkaar uitoefenen. Hetzelfde is het geval, wanneer een der deelstukken (gewoonlijk het kleinste) opnieuw met het andere in contact treedt, en zich op kosten van dit laatste voedt. Zeer dikwijls bezit deze invloed voor het voedende deelstuk een schadelijk karakter. Uit dit oogpunt beschouwd, laten zich

wellicht de eigenaardigheden der ziekelijke nieuwvormingen: proliferatie, metastasen, cachexie, als vermenigvuldigingsverschijnselen verklaren. Is deze zienswijze juist, dan zou daaruit misschien de gevolgtrekking gemaakt kunnen worden, dat de tumoren te bestrijden waren door te trachten dezen vermenigvuldigingsdrang te beteugelen. Op hoedanige wijze dat zou kunnen geschieden, vermag ik niet te beoordeelen. De gedachte kwam bij mij op of wellicht de lichaamssappen van steriele hybriden of van arbeidster-bijen op den blijkbaar overprikkelden vermeerderingsdrang een verdoovenden invloed zouden kunnen uitoefenen.

Zeker echter lijkt het mij, dat ziekelijke nieuwvormingen berusten op de aanwezigheid, in alle deelen van elk organisme, van een sluimerenden aanleg tot vermenigvuldiging, die echter slechts wakker wordt onder zekere, meestal schadelijke, invloeden (uit- zoowel als inwendige), en zich het meest openbaart op enkele bepaalde plaatsen, evenals de kiemvorming gewoonlijk gelocaliseerd is in bepaalde organen.

ZUSAMMENFASSUNG.

Fortpflanzung geschieht immer durch Teilung der Organismen in zwei oder mehr Stücke.

Zwischen geschlechtlicher und ungeschlechtlicher Fortpflanzung besteht kein wirklicher Unterschied. Ebensowenig ist die Fortpflanzung der einzelligen Wesen verschieden von derjenigen der Mehrzelligen. WEISMANN's Ansichten von der potentiellen Unsterblichkeit der Einzelligen, und der sekundären Entstehung der senilen Involution, sind nicht haltbar. Zwischen dem s.g. natürlichen und dem artifiziellen Tode lässt sich keine Grenze ziehen. Bei jedem Organismen-tod spielen sowohl innere wie äussere Einflüsse eine Rolle. Nicht die Frage: ob bei dem Aufhören des persönlichen Daseins eines bestimmten Individuums eine Leiche auftritt oder nicht, ist wichtig, sondern die Tatsache, dass alle Organismen ohne Ausnahme sterben können. Diese Fähigkeit ist eine Grundeigenschaft der lebenden Wesen. Durch sie wird eine, bisjetzt unüberschreitliche, Schranke dargestellt zwischen der lebenden und der leblosen Welt.

Die Stücke, worin sich ein Organismus bei der Fortpflanzung teilt, sind unter sich immer ungleich. Der Grad der Verschiedenheit kann alle möglichen Werte erreichen. Ist der Unterschied sehr gross, so erscheint das kleinste und am wenigsten entwickelte Stück als das Junge des grösseren und mehr ausgewachsenen, welches dadurch den Charakter eines Elter-organismus annimmt. Daraus geht aber kein Grund hervor um ein wesentlicher Unterschied anzunehmen zwischen Fortpflanzung durch Teilung (Somatogonie) und Fortpflanzung durch Keimzellen (Cytogonie). Es ist deshalb auch nicht richtig, zwei Arten von Plasma: Somatisches und Keim-plasma, einander gegenüberzustellen.

Die geschlechtliche Fortpflanzung beruht auf der Zusammenwirkung zweier verschiedenen Lebens-erscheinungen, die sich diametral gegenüber-

stehen. Es sind dies die eigentliche Fortpflanzung durch Teilung, und die Verschmelzung zweier (oder mehr) Individuen derselben oder sehr nahe verwandter Arten. Diese beiden Lebensäusserungen sind ursprünglich von einander unabhängig. Die sich vereinigenden Organismen können die gleiche Gestalt, Grösse und Lebensweise besitzen, oder sich in allen diesen Hinsichten so weit wie nur möglich von einander unterscheiden. Aus ihrer Verschmelzung geht ein Doppelorganismus hervor, dessen Fähigkeit zu der Fortpflanzung-durch-Teilung erhöht ist. Bei den Einzelligen führt die Konjugation also zur Steigerung des Vermehrungsdranges, bei den Mehrzelligen zur Keimesfurchung. Dass das Endresultat in diesem letzten Falle die Bildung eines einzigen Organismus ist, beruht auf der abermaligen Zusammentretung der ursprünglich selbständigen Furchungskugeln.

Man kann also auch umgekehrt behaupten: nicht nur alle Furchung sondern selbst alle Zellteilung ist ihrem Wesen nach Fortpflanzung. Von diesem Gesichtspunkt aus betrachtet, darf auch die abnormale Zellvermehrung als Fortpflanzungsvorgang aufgefasst werden. Diese Anschauung führt zu der Frage, ob sich die Tumorenbildung nicht bekämpfen liesse durch solche Mittel, welche die Fortpflanzung lähmen. Dabei könnte man denken an Sera aus unfruchtbaren Organismen, z.B. sterilen Hybriden, oder Arbeiterinnen der Bienen.

Physiology. — *The revival of the heart by means of soft Röntgenrays, after its arrest through the removal of its diffusible Potassium.* By H. ZWAARDEMAKER and T. P. FEENSTRA.

(Communicated at the meeting of December 21, 1929).

A heart after its arrest through the removal of its diffusible potassium can be revived by means of any corpuscular radiation (alpha or beta) of the right intensity (with a threshold of 0.8 radiumelement filtered only through mica or thin glass), but not by electromagnetic radiation ¹⁾. Since however the biological effect of the Röntgen-rays is generally attributed to the rapidly moving electrons deliberated by the rays in the tissues, we tried to obtain an analogous result by means of increasing the absorbibility of the rays by diminishing their hardness ²⁾. Not before this year under the directory of Prof. NOYONS we were able to surmount all the technical difficulties we met with and could we obtain a complete success.

The isolated heart of an eel or of a frog was placed before a Röntgentube with LINDEMANN window (permeable to soft rays) at a distance of 5 millimeter. The organ, armed in the case of the eel with a sinuscanule, in that of the frog with a KRONECKER canule, was permanently perfused, firstly during $\frac{1}{4}$ to $1\frac{1}{4}$ hours with normal Ringersolution, then with Ringersolution without potassium. When the heart had come to a critical standstill without grouping it was radiated during 5 times 5 minutes (with pauses of 5 minutes between the expositions) by soft Röntgenrays, created by a secondary current of 10 milliamperes at a potential of 9000 Volts measured on a statical Voltmeter. The glowcurrent measured 4.5 Ampere. The radiant heat coming from the tube, when working was of no influence, as controlling experiments proved.

If we do not count the preparatory experiments performed before, and only pay attention to a technically perfect series of 14 experiments, performed in recent days, we found :

no revival	0 times
a somewhat dubious revival	6 „
a perfect revival	8 „

When we put together 10 complete experiments with a perfect revival we calculated the following average values :

the time elapsing between the moment of the critical arrest and the beginning of the Röntgen-radiation	10 minutes
the latency counted from the beginning of the radiation to the beginning of the pulsation	28 „
the duration of the full and regular pulsations	6 hours

¹⁾ Ned. Tijdschr. v. Geneesk. 1916, II. N^o. 22.

²⁾ Strahlentherapie, Bd. 29, p. 527. 1928.

The result was attained by the sacrifice of 4 milliampere-hours in the tube whilst earlier ¹⁾ the same was obtained by means of cathode rays with 0.25 milliampere-hours.

The Röntgen-rays we used belong to the BUCKY zone and give an erytheme dosis in 10 minutes.

Physiol. Lab.

Utrecht, 14 December 1929.

¹⁾ Verslag Kon. Akad. v. Wetensch. Amsterdam, June 27, 1925.

H. ZWAARDEMAKER AND T. P. FEENSTRA: THE REVIVAL OF
THE HEART BY MEANS OF SOFT RÖNTGENRAYS, AFTER ITS ARREST
THROUGH THE REMOVAL OF ITS DIFFUSIBLE POTASSIUM.

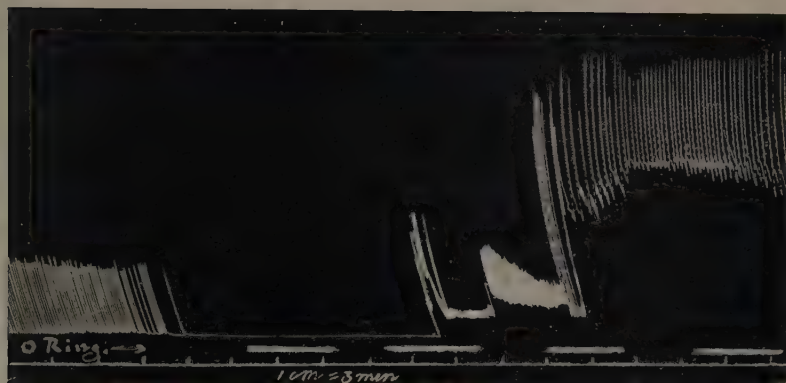


Fig. 1.
Revival of a frog heart by means of soft Röntgenrays.
— — Periodes of radiation
1 cm. = 3 min. = unit of time.

(Communicated at the meeting of December 21, 1929.)

Osmosis of binary liquids through a membrane $M(n)$.

We now consider an osmotic system



in which L and L' are binary liquids; we imagine them represented by two points on the line WX of figs. 1–4 M.O.I. For the sake of concentration we shall suppose the liquid L to be always on the left side of L' in these figures. As membrane $M(n)$ is permeable for all substances, we may imagine the four D.T.'s (Diffusion-Types) of scheme I and the four transition-D.T.'s of scheme II.

Scheme I			Scheme II		
W		X	W		X
a.	← 0	← (cond.)	e.	—	←
b.	→	←	f.	→	—
c.	→	→ 0 (cond.)	g.	[—	→ 0]
d.	[← 0	→ 0]	h.	[← 0	—]

Previously ¹⁾ we have deduced:

in an osmotic system with a membrane $M(n)$ all D.T.'s are possible, except the incongruent one.

In order to determine the incongruent D.T. we must therefore begin by knowing the incongruent direction of the water and of the substance X in system (1). Previously we have said:

the congruent direction of a substance is the direction in which it travels through a membrane, permeable for this substance only; the incongruent direction is the opposite one.

In order to determine the congruent direction of the water, we have to replace the membrane $M(n)$ in system (1) by a membrane $M(W)$; we then have:



in which the water diffuses according to the direction of the arrow;

¹⁾ F. A. H. SCHREINEMAKERS. Verslagen Kon. Akad. v. Wet. Amsterdam, 37 634 (1928); These Proceedings, 31 811 (1928).

this follows at once from our discussions in M.O. III, e.g. from F , G or H .

In order to determine the congruent direction of the substance X we take the system.

$$L \mid M(X) \mid L' \longleftarrow X \dots \dots \dots (3)$$

Now we find that the substance diffuses towards the left.

We now know the congruent directions of W and X in (2) and (3); we now see that the arrows, beside which the sign 0 has been placed in schemes I and II, indicate an incongruent direction; so b is the congruent and d the incongruent D.T.; a and c are mixed D.T.'s. From this follows:

in system (1) the three D.T.'s a , b and c and the two transition-D.T.'s e and f are possible; the D.T.'s placed between parentheses, are not possible.

We now suppose the compositions of the liquids L and L' of system (1) kept constant in some way or other, so that a stationary current of W and X travels through the membrane.

We now represent the quantities of W and X , diffusing in a definite time and through a definite part of the surface of a membrane according to the congruent D.T. (viz. b) by w_b and x_b .

The more the membrane now approaches a membrane $M(X)$ and the quantity of W , which diffuses, consequently decreases, the more $w_b : x_b$ will approach zero.

The more the membrane approaches a membrane $M(W)$ and the diffusing quantity of X consequently decreases, the more $w_b : x_b$ will approach ∞ ; so we find:

A) in the congruent D.T. the ratio between the diffusing quantities of W and X can have all values between 0 and ∞ .

This does not obtain any longer, however, in the mixed D.T.'s a and c . We imagine a membrane, for which the D.T. a obtains; we shall call it a membrane $M(a)$. We assume that w_a quantities of W and x_a quantities of X diffuse through it; we shall represent this diffusion by:

$$M(a) \longleftarrow 0 w_a \text{ mol. } W \longleftarrow x_a \text{ mol. } X \dots \dots (4)$$

We now imagine also a membrane $M(c)$ for which the D.T. c obtains; we represent this diffusion by:

$$M(c) \longrightarrow w_c \text{ mol. } W \longrightarrow 0 x_c \text{ mol. } X \dots \dots (5)$$

We now imagine these membranes $M(a)$ and $M(c)$ between the liquids L and L' at the same time; then through the one membrane W and X will run towards the left and through the other towards the right.

We now regulate the surfaces of these membranes in such a way, that in the same period of time as much water will go through $M(c)$ towards the right as through $M(a)$ towards the left. We can do this

by taking the surface of the membrane $M(c)$ $w_a : w_c$ times larger; then the diffusion through the membrane $M(c)$ will become :

$$M(c) \longrightarrow w_a \text{ mol. } W \longrightarrow 0 \frac{w_a}{w_c} \cdot x_c \text{ mol. } X. \quad (6)$$

If the diffusions according to (4) and (6) now take place at the same time, the W -quantity of both liquids will not change; the quantity of X , however, does change; for now we still have the diffusion :

$$\longrightarrow 0 \left(\frac{w_a}{w_c} \cdot x_c - x_a \right) \text{ mol. } X. \quad (7)$$

We now distinguish three cases :

1) $\frac{w_a}{w_c} x_c > x_a$. Now a part of the substance X will diffuse towards the right, viz. incongruently. If we now put in still an other membrane $M(X)$, then by regulating its surface, we can arrange that the same quantity of X runs towards both sides. As we then have a system, in which the liquids change neither their compositions nor their quantities, we should get eternal circular currents of W and X ; we assume however that this is not possible.

2) $\frac{w_a}{w_c} x_c = x_a$. Now the same quantity of W and the same quantity of X runs in both directions; then we also have continuous circular currents of W and X .

3) $\frac{w_a}{w_c} x_c < x_a$. The substance X now runs in congruent direction; as we cannot neutralize this current by an other membrane, both liquids will change; so at last a state of equilibrium will arise, so that the X -current disappears also.

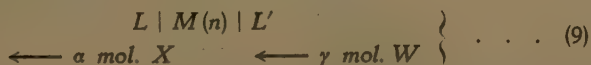
From this follows that only case 3 can be possible, viz.

$$\frac{w_a}{w_c} < \frac{x_a}{x_c} \quad (8)$$

So we may say :

B) in each of the mixed D.T.'s the ratio of the diffusing quantities of W and X can have an infinite number of values; these values, however, must always be smaller in D.T. a than in D.T. c.

We may further elucidate the previous considerations with the aid of the O.W.A. and the O.X.A. of the liquids. For this purpose we suppose an osmotic system :



in which α mol. X and γ mol. W diffuse towards the left. When α and

γ are very small, then, as we have seen before¹⁾, they have to satisfy:

$$a [\xi_x - \xi'_x] + \gamma [\xi_w - \xi'_w] > 0 \quad . \quad . \quad . \quad . \quad . \quad (10)$$

in which ξ_x and ξ'_x represent the O. X. A., and ξ_w and ξ'_w the O. W. A. of the liquids L and L' . We now represent the X -amount of these liquids by x and x' (consequently their W -amount by $1-x$ and $1-x'$). As we shall see later on we now can prove that a and γ must satisfy:

$$a [1 - (x)] - \gamma (x) > 0 \quad . \quad . \quad . \quad . \quad . \quad (11)$$

in which (x) has a definite value, which is situated between x and x' . If we take $x < x'$, as we have always supposed it to be until now, then we consequently have:

$$x < (x) < x' \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

We now begin by supposing a membrane $M(W)$ in system (9), so that no X diffuses; then we have the system (2), discussed already before. As now $a=0$, γ must satisfy:

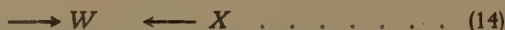
$$-\gamma (x) > 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

as follows from (11); therefore γ must be negative. This means, as it appears from (9) that the water must diffuse towards the right. This is in accordance with the arrow in system (2).

If in (9) we imagine a membrane $M(X)$, we get system (3). As now $\gamma=0$, it follows from (11) that a must be positive. (9) shows that X must now diffuse towards the left; this is also in accordance with the arrow in system (3).

In accordance with previous discussions it appears from this that in scheme I b is the congruent D.T., d the incongruent and a and c are the mixed D.T.'s.

(11) can always be satisfied by pos. values of a and neg. values of γ ; it appears from (9) that the diffusion then takes place according to the D.T.:



viz. the congruent D.T. of scheme I; what has been said in A above, also follows from it.

If we were to take $\gamma=\text{pos.}$ and $a=\text{neg.}$, we should have the incongruent D.T.:



This is not possible, however, because the first part of (11) for $\gamma=\text{pos.}$ and $a=\text{neg.}$ is always negative.

We can also satisfy (11) by pos. values of γ and a ; then we have the mixed D. T. a of scheme I. We now cannot take a and γ arbitrarily, however; for, as appears from (11) they must satisfy:

$$\frac{\gamma}{a} < \frac{1 - (x)}{(x)} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

¹⁾ F. A. H. SCHREINEMAKERS, l. c.

From this it appears that in the D.T. *a* the ratio of the diffusing quantities of *W* and *X* must always be smaller than the value of the second part of (16).

(11) can also be satisfied by negative values of α and γ ; we then have the mixed D.T. *c*. If we put $\alpha = -\alpha'$ and $\gamma = -\gamma'$ so that α' and γ' are positive, then we find from (11) that they have to satisfy:

$$\frac{\gamma'}{\alpha'} > \frac{1-(x)}{(x)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

So in the D.T. *c* the ratio of the diffusing quantities of *W* and *X* must always be greater than the value of the second part of (17).

Consequently from (16) and (17) follows not only what has already been said above in *B*, but besides that the limit of these ratios is determined by the second part of (16) and of (17).

We now suppose the liquids *L* and *L'* represented once more by two points of the line *WX* (figs. 1–4 M.O. I); the liquid *L* (as has been frequently stated before) is always situated on the left side of *L'*.

In the D.T. *b* the liquid *L* gives off water and takes in *X*; both reasons cause the *X*-amount of *L* to increase; consequently *L* travels along line *WX* towards the right.

As liquid *L* takes in water and gives out *X*, its *W*-amount will consequently decrease of these two reasons; so *L'* moves towards the left.

We may represent this by:

$$D.T. \, b \quad \longrightarrow L \quad \longleftarrow L' \quad . \quad . \quad . \quad . \quad . \quad (18)$$

in which the arrows indicate the direction in which the liquids travel along line *WX*.

In the mixed D.T. *a*, however, we may distinguish two cases, namely:

$$D.T. \, a \quad \longrightarrow L \quad \longleftarrow L' \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$D.T. \, a \quad \longrightarrow L \quad \longrightarrow L' \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The liquid *L* namely absorbs water and *X*; now we may say that liquid *L* absorbs a liquid *L*₀; this liquid *L*₀ contains:

$$\alpha \text{ mol. } X + \gamma \text{ mol. } W \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The liquid *L'* now gives out this liquid *L*₀. As α and γ are both positive, we now may also write for (11):

$$\frac{\alpha}{\alpha + \gamma} > (x) \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

in which the first part is the *X*-amount of the liquid *L*₀. If we represent the *X*-amount of *L* and *L'* by x and x' , then it follows from the signification of (x):

$$x < (x) < x' \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

This shows that α and γ will most certainly satisfy :

$$\frac{\alpha}{\alpha + \gamma} > x \quad (24)$$

in which the second part is still smaller than in (22). This means :

the diffusing liquid L_0 has a larger X -amount than L . As, therefore, L absorbs a liquid with a larger X -amount, L will consequently shift towards the right.

If in (22) we substitute (x) by the larger value x' , then the first part may be larger as well as smaller than the second; so we have :

$$\frac{\alpha}{\alpha + \gamma} \geq x' \quad (25)$$

This means: the diffusing liquid L_0 may have a greater as well as a smaller and also a similar X -amount as the liquid L' .

From this it appears that the liquid L' may travel as well towards the left as towards the right and may also happen to remain unchanged.

With the osmosis according to the mixed D.T.c we may distinguish two corresponding cases, which the reader will be able to deduce easily; then the liquid L' will always shift towards the left.

Above we have seen that α and γ in system (9) have to satisfy (10). Here, as we have seen formerly:¹⁾

$$\xi_x = -\zeta - (1-x) \frac{\partial \zeta}{\partial x} ; \quad \xi_w = -\zeta + x \frac{\partial \zeta}{\partial x} \quad (26)$$

We now have :

$$\xi'_x = \xi_x + \int_x^{x'} d\xi_x \quad \xi'_w = \xi_w + \int_x^{x'} d\xi_w \quad (27)$$

From (26) follows :

$$d\xi_x = -(1-x)r dx \quad d\xi_w = xr dx \quad (28)$$

in which the meaning of r follows from the deduction. If we substitute these values in (27), we find :

$$\xi_x - \xi'_x = \int_x^{x'} (1-x)r dx = \int_x^{x'} r dx - \int_x^{x'} xr dx \quad (29)$$

$$\xi_w - \xi'_w = - \int_x^{x'} xr dx \quad (30)$$

¹⁾ 1. c.

As $x' > x$ and r is always positive, these integrals are positive also. It is easy to see now that we may put:

$$\int_x^{x'} xr \, dx = (x) \int_x^{x'} r \, dx \quad (31)$$

in which (x) has a definite value between x and x' . If we substitute this in (29) and (30), we get:

$$\xi_x - \xi'_x = [1 - (x)] \int_x^{x'} r \, dx \quad (32)$$

$$\xi_w - \xi'_w = -(x) \int_x^{x'} r \, dx \quad (33)$$

If we substitute this value in (10) and if we divide by the integral, we find (11).

We now imagine a system

$$\left. \begin{array}{l} n \times L \mid M(n) \mid n' \times L' \\ \longleftarrow \Delta a \text{ mol. } X \quad \longleftarrow \Delta \gamma \text{ mol. } W \end{array} \right\} (34)$$

in which $\Delta a \text{ mol. } X$ and $\Delta \gamma \text{ mol. } W$ diffuse towards the left. We then get the system:

$$(n + \Delta a + \Delta \gamma) \times L_1 \mid M(n) \mid (n' - \Delta a - \Delta \gamma) \times L'_1 \quad . . . (35)$$

in which L_1 and L'_1 represent the new liquids. The first system has a thermodynamical potential $n\zeta + n'\zeta'$; that of the second system is

$$(n + \Delta a + \Delta \gamma) \zeta_1 + (n' - \Delta a - \Delta \gamma) \zeta'_1$$

If we now put $\Delta a + \Delta \gamma = \Delta s$ then, as the thermodynamical potential may decrease only, we must have:

$$(n + \Delta s) \zeta_1 + (n' - \Delta s) \zeta'_1 - n\zeta - n'\zeta' < 0 \quad . . . (36)$$

We represent the increase of the X -amount of the left-side liquid by Δx ; we then have:

$$\Delta x = \frac{nx + \Delta a}{n + \Delta a + \Delta \gamma} - x = \frac{\Delta a - x \Delta s}{n + \Delta s} \quad (37)$$

If we represent the increase of the X -amount of the right-side liquid by $\Delta x'$, then we have:

$$\Delta x' = \frac{n'x' - \Delta a}{n' - \Delta a - \Delta \gamma} - x' = \frac{-\Delta a + x' \Delta s}{n' - \Delta s} \quad . . . (38)$$

Now we write (36) in the form:

$$(n + \Delta s)(\zeta_1 - \zeta) + (n' - \Delta s)(\zeta'_1 - \zeta') + \Delta s \cdot (\zeta - \zeta') < 0 \quad (39)$$

If we substitute herein the values of $n + \Delta s$ and $n' - \Delta s$, which follow from (37) and (38), then (39) passes into:

$$(\Delta a - x \Delta s) \frac{\zeta_1 - \zeta}{\Delta x} - (\Delta a - x' \Delta s) \frac{\zeta'_1 - \zeta'}{\Delta x'} + \Delta s (\zeta - \zeta') < 0 \quad (40)$$

We now suppose the ζ -curve of the liquids, consisting of W and X , drawn in an x, ζ -diagram. We imagine a line through the two points, representing the thermodynamical potentials of the two left-side liquids L and L_1 (systems 34 and 35); we call this line the left-side chord. As the coordinates of the one point are x and ζ and those of the other point $x + \Delta x$ and ζ_1 , so for the coordinates Z and X of this chord obtain:

$$Z = \zeta + \frac{\zeta_1 - \zeta}{\Delta x} (X - x) \quad (41)$$

Now we also imagine a line through the two points, representing the thermodynamical potentials of the two right-side liquids L' and L'_1 (systems 34 and 35). For this right-side chord now obtains:

$$Z = \zeta' + \frac{\zeta'_1 - \zeta'}{\Delta x'} (X - x') \quad (42)$$

These left-side and right-side chords intersect one another in a point; if we represent the X of this point by (x) , then this satisfies:

$$\zeta + \frac{\zeta_1 - \zeta}{\Delta x} [(x) - x] = \zeta' + \frac{\zeta'_1 - \zeta'}{\Delta x'} [(x) - x'] \quad (43)$$

We now write (42) in the form:

$$[(x) - x] \frac{\zeta_1 - \zeta}{\Delta x} - [(x) - x'] \frac{\zeta'_1 - \zeta'}{\Delta x'} + (\zeta - \zeta') = 0 \quad (44)$$

We now multiply (43) by Δs and then subtract this from (40); we then get:

$$[\Delta a - (x) \Delta s] \frac{\zeta_1 - \zeta}{\Delta x} - [\Delta a - (x) \Delta s] \frac{\zeta'_1 - \zeta'}{\Delta x'} < 0 \quad (45)$$

or:

$$[\Delta a - (x) \Delta s] \left[\frac{\zeta_1 - \zeta}{\Delta x} - \frac{\zeta'_1 - \zeta'}{\Delta x'} \right] < 0 \quad (46)$$

We now have:

$$\frac{\zeta_1 - \zeta}{\Delta x} < \frac{\zeta'_1 - \zeta'}{\Delta x'} \quad (47)$$

The left-side form namely is the tangens of the angle, which the

left-side chord makes with the X -axis; the right-side form is the tangens of the angle, which the right-side chord makes with the X -axis. In a system in which no unmixing can occur and in which, therefore, every liquid is stable with respect to all other liquids, the tangens of this right-side angle is always greater than the tangens of the left-side angle. We see this at once when we keep in mind that the ζ -curve is convex in all points. As the second factor of (45) is, therefore, negative, the first must be positive; from this follows:

$$\Delta\alpha - (x)\Delta s > 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

If we here put $\Delta s = \Delta\alpha + \Delta\beta$ again, then (47) passes into:

$$\Delta\alpha \cdot [1 - (x)] - \Delta\gamma(x) > 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

Above we have seen that (x) is determined by the point of intersection of the left-side and right-side chords in the x, ζ -diagram. If we imagine (x) represented by a point on the line WX , then we see that the left-side liquids L and L_1 are situated on the left side of (x) and the right-side liquids L' and L'_1 on the right side. From this follows:

$$x \text{ and } x + \Delta x < (x) < x' \text{ and } x' + \Delta x' \quad . \quad . \quad . \quad . \quad (50)$$

It appears from the deduction that $\Delta\alpha$ and $\Delta\gamma$ may be finitely as well as infinitely small; if we take both infinitely small and if we put $\Delta\alpha = \alpha$ and $\Delta\gamma = \gamma$, then (48) passes into (11). As Δx and $\Delta x'$ are now infinitely small also, the left-side and the right-side chords will pass into tangents to the ζ -curve; then (x) is determined by the point of intersection of the left-side and right-side tangents; so in (11) (x) has an other value than in (48).

We shall further refer to (48) later on.

(To be continued).

Leiden, Lab. of Inorg. Chemistry.

Physics. — *New measurements on the vapour pressure curve of liquid helium.* II. By W. H. KEESOM, SOPHUS WEBER and G. SCHMIDT. (Communication N^o. 202c from the Physical Laboratory at Leiden.)

(Communicated at the meeting of December 21, 1929).

§ 1. *Introduction.* In continuation of the measurements on the vapour pressure curve of liquid helium in the pressure range from 76 to 1 cm Hg ¹⁾, we have now performed measurements down to lower pressures. As lowest pressure we reached about 0.005 cm Hg.

In this range the thermomolecular pressure difference attains to considerable importance, particularly in the helium thermometers, in which the pressure becomes very small. Although an extensive research on the thermomolecular pressure difference has not yet been performed, we now have available data, which determine the behaviour of this pressure difference with sufficient certainty for our purpose.

These measurements on the vapour pressure consist of 4 series. The first two series are a continuation of the measurements of Dec. 1927 and Jan. 1928 of Comm. N^o. 202b, and extend them to ca. 0.1 cm Hg ²⁾. The last two series refer to the lowest vapour pressures obtainable.

§ 2. *The thermometers.* For these measurements we employed the same thermometers I, II and III, which are described in detail in Comm. N^o. 202b; also the Wollaston hot wire manometers by which the pressure in the thermometers was measured.

Thermometer I has a narrow capillary; this gives a small noxious volume, but causes a large thermomolecular pressure difference, whereas for thermometer III with a wide capillary, the opposite is the case.

The diameters of the capillaries of Therm. I, II and III are respectively 0.2029 cm, 0.2096 cm and 0.3694 cm.

Therm. II has no bulb, and serves for elimination of the unknown temperature distribution along the capillaries of therm. I and III.

The correction for the noxious volume, which must be determined accurately, amounts for Therm. I and III, respectively to ca. 1.5 and ca. 4 % at the boiling point, but becomes continually smaller at lower temperatures.

Fig. 3 of Comm. N^o. 195c ³⁾ shows how these thermometers are placed in the cryostat. The bulbs of the two thermometers are indicated by Th_1 and Th_2 ; Ma_1 , Ma_2 and Ma_3 are the hot wire manometers, surrounded by tubes filled with vaseline oil, immersed in an ice bath.

In each series of measurements we always began by determining the

¹⁾ W. H. KEESOM, SOPHUS WEBER and G. NØRGAARD. These Proceedings 32, 864, 1929; Comm. Leiden N^o. 202b.

²⁾ These measurements were already mentioned in Comm. N^o. 202b § 6.

³⁾ These Proceedings 32, 710, 1929; Comm. Leiden N^o. 195c.

boiling point of liquid helium. The influence of the change in the calibration of the manometers can now be eliminated by shifting the curve, obtained after having corrected for the thermomolecular pressure difference, so as to pass through the boiling point, $T = 4.22^\circ \text{ K.}$, that has already been fixed. This shifting, that was already discussed in Comm. N^o. 202*b*, is generally less than 2 % ; it is due in particular to changes in the gas film that covers the wire.

The corrections on account of the deviation of helium from the Avogadro state may be neglected in this investigation also.

According to KAMERLINGH ONNES and BOKS¹⁾ for helium $B_{A\ 4.22^\circ \text{ K}} = -0.00007$, whereas we put, extrapolating, $B_{A\ 1.00^\circ \text{ K}} = -0.00009$.

The correction has a maximum in tables II and III. We find at $T = 4.22^\circ \text{ K.}$ and $T = 1.00^\circ \text{ K.}$ respectively

$$\frac{T_{\text{corr. B}}}{T} = 1 + 0.0005,$$

$$\frac{T_{\text{corr. B}}}{T} = 1 + 0.002.$$

§. 3. *The correction for the thermomolecular pressure difference.* As already stated in § 1, the thermomolecular pressure difference between the hot wire manometer, in ice, and the thermometer bulb, immersed in liquid helium, becomes of great importance for the determination of the lowest helium temperatures.

Whereas the correction for the noxious volume can almost be neglected under these circumstances, the thermomolecular pressure difference increases in such measure, that we may hardly speak of a correction here. This also appears from the fact, that, if it were possible to reach the limiting condition $p_1/p_2 = \sqrt{T_1/T_2}$, one should obtain for $T_1 = 273.1$ and $T_2 = 1.0$: $p_1/p_2 = 16.55$.

On account of the large decrease in the mean free path, with decreasing temperature however, it is almost impossible to make $2R/\lambda$ small at the temperature of liquid helium. Thus we obtain a state, in which $2R/\lambda$ is very small in the higher part of the thermometer capillary, and is very large in the lower part, where the capillary is immersed in liquid helium. As a consequence of this, it is of great importance for future researches at these temperatures, that the behaviour of the thermomolecular pressure difference be investigated. Indeed for all investigations, at which low pressures occur and are measured in a part of the apparatus that has a temperature different from that of the space where the pressure is to be known, this correction must be taken into account. Accordingly we have already commenced this research by the method, briefly described in Comm. N^o. 202*b*. Some preliminary measurements in order to study the method have already been made.

¹⁾ C. A. CROMMELIN, Comm. Leiden Suppl. N^o. 63, p. 21.

²⁾ H. KAMERLINGH ONNES and SOPHUS WEBER. These Proceedings 18, 1493, 1915. Comm. Leiden N^o. 147*b*.

In Comm. N^o. 147b²⁾ the theoretical behaviour of the thermomolecular pressure difference in the neighbourhood of the two limiting conditions, $p_1/p_2 = \sqrt{T_1/T_2}$ and $p_1 = p_2$, is explained and based upon these calculations an attempt is made to give a formula for the whole pressure range. For lack of enough experimental data, there could then be given only roughly estimated values for the constants of this formula.

The formula proposed there is as follows¹⁾:

$$\frac{dp}{p} = \frac{1}{2} \left(\frac{1 + c_1 c_2 \frac{2R}{\lambda}}{1 + c_1 \frac{2R}{\lambda}} \right) \cdot \frac{1}{\left(1 + \frac{2R}{\lambda}\right) \left(1 + c_3 \frac{2R}{\lambda}\right)} \frac{dT}{T}, \quad \dots (1)$$

where λ is the mean free path, according to O. E. MEYER²⁾, which for helium may be derived from:

$$p\lambda = 28.31 \left(\frac{T}{T_0} \right)^{1+n} = 28.31 \left(\frac{T}{T_0} \right)^{1.15}, \quad \dots (2)$$

when the viscosity of helium η is determined by $\eta/\eta_0 = (T/T_0)^{1/2+n}$, putting $n = 0.15$ ³⁾, as was found in Comm. N^o. 134b⁴⁾.

From the formulae (1) and (2) it follows that the thermomolecular pressure difference for a definite temperature difference is entirely determined by the value of $2R/\lambda$, so that one can easily determine the thermomolecular pressure difference for another value of R , when it is known of one diameter $2R$.

For our preliminary measurements of the thermomolecular pressure difference we have performed two series of measurements: one, at which the thermometer bulb was immersed in liquid oxygen, $T_2 = 90.36^\circ \text{K.}$, and one, at which it was immersed in liquid hydrogen, $T_2 = 20.41^\circ \text{K.}$, on both occasions the hot wire manometer being maintained at 0°C. These measurements were performed with thermometer N^o. I (Comm. N^o. 202b), thus $2R = 0.2029 \text{ cm.}$

The results of these measurements are presented in Fig. 1 (the points are indicated by \odot).

¹⁾ MARTIN KNUDSEN, Kgl. Danske Videnskab. Selskabs Meddelelser 8, 3, 1927, has given in his investigation on the thermomolecular pressure difference of hydrogen a somewhat modified formula. We will return to this in our next research, also in connection with the behaviour in the neighbourhood of the limiting conditions.

²⁾ O. E. MEYER, Die kinetische Theorie der Gase, Breslau, p. 189, 1899. Though at present it must be considered better to calculate the mean free path according to S. CHAPMAN, Phil. Trans., A 216, 279, 1915, we will use in this communication the mean free path according to O. E. MEYER; also because λ is here only an auxiliary parameter, which disappears out of the result.

³⁾ At least to 20°K. As however, as already said in Comm. N^o. 147b, p. 28, the thermometer corrections are almost entirely due to that part of the capillary that has higher temperatures than 20°K. , we may assume for these calculations, that this formula is available for the whole temperature range investigated.

⁴⁾ H. KAMERLINGH ONNES and SOPHUS WEBER. These Proc. 15, 1396, 1913.

The abscissa is $\log_{10} p_1$, where p_1 means the pressure of the hot wire manometer, measured directly; as ordinate we took $\log_{10} (1 - p_2/p_1)$.

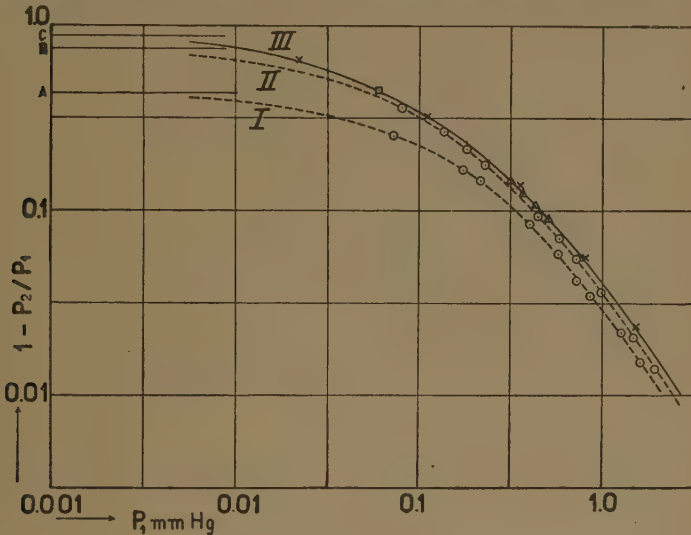


Fig. 1.

where p_2 means the pressure in the thermometer bulb. The curves I and II, drawn through the points measured, have as asymptotes on the low-pressure side, the straight lines A and B: $1 - p_2/p_1 = 1 - \sqrt{T_2/T_1} = 0.425$, and 0.727 respectively. So the curve that corresponds to the helium temperature $T_2 = 4.22^\circ \text{ K}$. must have as asymptote the straight line C, for which $1 - p_2/p_1 = 0.875$.

Integration of (1), taking into account (2); when putting:

$$y = \frac{2R}{\lambda}, \text{ and } c_1 = 0.30, \quad c_2 = 1.70, \quad c_3 = 0.12^1),$$

gives

$$\frac{T_1}{T_2} = \left(\frac{y_1 + 9.322}{y_2 + 9.322} \right)^{0.0188} \cdot \left(\frac{y_1 + 0.609}{y_2 + 0.609} \right)^{0.6489} \cdot \left(\frac{y_1 + 2.810}{y_2 + 2.810} \right)^{0.00157} \cdot \left(\frac{y_1}{y_2} \right)^{-1.539} \quad (3)$$

By means of this equation we obtain for $T_1 = 273.1$ and $T_2 = 4.22$ the following corresponding values for p_1 and p_2 (for $2R = 0.2029 \text{ cm.}$):

p_2	p_1	p_1/p_2
10 bar	29.0 bar	2.90
100 "	147.0 "	1.47
400 "	464 "	1.16
1000 "	1060 "	1.06
2000 "	2040 "	1.02

¹⁾ In Comm. N^o. 147b, p. 28, line 8 from the top, the values c_1 and c_2 , owing to a printer's error, are incorrect and should be $c_1 = 0.3101$ and $c_2 = 2.865$.

In Fig. 1 these values are indicated by the points \times . The curve III, drawn through these points, appears to have the same course as the two experimental curves I and II.

As a consequence of the limiting condition ¹⁾

$$p_1^2 - p_2^2 = c(T_1^{2.30} - T_2^{2.30})$$

the three curves must, for large values of p_1 , approach asymptotically to a straight line of the form :

$$\log(1 - p_2/p_1) + 2 \log p_1 = \text{const.},$$

which is indeed pretty well the case.

In order to control the curve III, which we must use for the corrections for the thermomolecular pressure difference, we have repeated some series of measurements with the thermometers I and III at the helium temperatures known from Comm. N^o. 202*b*, but this time with a very low ice point pressure. This gave a direct determination of the thermomolecular pressure difference, and thus a direct experimental determination of some points of curve III.

For that purpose we have derived from the measurement on May 24th 1929 (table V) for thermometer I the indication, $T_{l.a.}$, at the boiling point of helium. The ratio of this to 4.22 gives immediately p_1/p_2 . This gives the point \square in Fig. 1. The uncertainty of this point, due to the change in the calibration of the hot wire manometer, can be estimated as ca. 1 %.

Further we chose four points from the first two measurements in table III. Comparison of the values of $T_{l.a}$ given in this table, with the temperatures given in Comm. N^o. 202*b*, particularly the temperatures obtained in the measurements of Dec. 19th, 1927, where the corrections for the thermomolecular pressure difference were very small, gave the four points on curve III, which are indicated by Δ .

These points also, agree very satisfactorily with the theoretical curve III, which gives sufficient justification for using this curve, and formula (3), for the calculation of the corrections for the thermomolecular pressure difference. As appears from the following, the temperatures measured with the thermometers I and III, and corrected with this method, agree very well with each other.

At the temperatures, measured by us, and the ice point pressures we used, there is generally no difference between the thermomolecular pressure difference within the temperature intervals 273.1—4.22° K. and 273.1—1.0° K. When however, with a view to reaching still lower temperatures, the ice point pressures are taken still lower, a thermomolecular pressure difference can also appear in the temperature interval 4.22—0.85° K., as formula (3) shows also. We must however consider then that the constants c_1 , c_2 and c_3 may have other values.

¹⁾ To be derived from equations (1) and (2) for large p 's.

§ 4. *The measurement of the vapour pressure.* For vapour pressures below 1 cm. Hg., these were measured :

a. in the completely closed bulb V_p (Fig. 3 of Comm. N^o. 195c) with a volume of ca. 3 cm³, and a capillary with a known diameter ; in it ca. 1 cm³. liquid helium was condensed. This helium stood at the same height as the middle of the thermometer bulb ;

b. in an open tube T_u (Fig. 3 of Comm. N^o. 195c), of which the lower part is immersed in the liquid helium of the bath. Here the vapour pressure is measured on the height of the level of the liquid in the bath, but in the equilibrium state of the liquid and the vapour, which occurs in the tube.

For the apparatus and the details we refer to Fig. 1 and 3 of Comm. N^o. 195c. The pressure in the closed vapour pressure tube V_p is measured by means of the accurately calibrated McLeod gauge Mc_2 by opening the valve K_7 ; K_1 and K_2 being closed. The pressure can also be measured by means of the hot wire manometer Ma_3 besides the McLeod gauge. The former served only for control in order to indicate small oscillations.

The pressure in the open tube T_u was measured by means of the same McLeod gauge, by closing K_7 and opening K_2 .

We could also measure the pressure in the exhaust funnel F by closing K_7 and K_2 and opening K_1 .

For further technical particulars we refer to Comm. N^o. 195c.

In the decision of our results we have preferred to employ the pressures measured with the method a, as in the method b there are some uncertainties. We may expect that, in spite of permanent stirring of the liquid, the uppermost liquid layer, in consequence of the quick evaporation, has a somewhat lower temperature than that measured by the two thermometers. This should be able to exert an influence in the tube T_u , in which the measurement of the vapour pressure will then be too low.

A deviation in the same direction can also appear, because at low pressures the hydrostatic pressure difference has an influence. By permanent stirring we prevent this difficulty for the most part, but perhaps not completely.

TABLE I.

p bulb V_p cm Hg	p open tube T_u cm Hg	p exhaust funnel F cm Hg
0.174	0.154	0.151
0.0218	0.0200	0.0178
0.0063	0.0055	0.0018
diameter capillary bulb V_p		0.298 cm
" open tube T_u		1.15 "
" cryostat glass		4.80 "

Neither difficulties occur with the method *a*. However it is then advisable to take the capillary thin walled and not too wide, in spite of the greater correction for the thermomolecular pressure difference that is thus caused. Too large a diameter and too great a mass of glass of the capillary would give rise to a danger of too great a heat transport to the liquid helium in the bulb, and thus of a temperature increase in it. The expectation that at lower pressures one would obtain too low results for the pressures with method *b*, is confirmed by Table I.

The difference between the pressures in the open tube and in the exhaust funnel is entirely in agreement with the difference calculated for the known pump capacity (675 liter sec).

§ 5. *The results of the measurements* are given in the tables II, III, IV, V and VI.

$T_{I,a}$ and $T_{III,a}$: temperatures as derived from the pressures observed in the thermometers, corrected for noxious volume and thermal contraction of the bulb.

$T_{I,b}$ and $T_{III,b}$: have been derived from the values of $T_{I,a}$ and $T_{III,a}$ by applying the thermomolecular pressure difference correction.

$T_{I,c}$ and $T_{III,c}$: have been derived from $T_{I,b}$ and $T_{III,b}$, after having shifted the temperature curve, so that the boiling point comes to $T = 4.22^\circ \text{K}$.

The vapour pressure was measured by method *a* discussed in § 4, and corrected for the thermomolecular pressure difference.

TABLE II.

Measurements on Febr. 1st 1929.

Ice point pressures:

Thermometers I and III: $p_0 = 5.575 \text{ cm Hg}$

„ II: $p_0 = 1.135 \text{ „ „}$

Vapour pressure $p \text{ cm Hg}$	$T_{I,a}$	$T_{III,a}$	$T_{I,b}$	$T_{III,b}$	$T_{I,c}$	$T_{III,c}$
77.27	4.419	4.327	4.214	4.261	4.237	4.237
35.12	3.682	3.578	3.480	3.512	3.498	3.492
4.130	2.399	2.288	2.194	2.213	2.206	2.201
3.557	2.338	2.225	2.131	2.150	2.143	2.138
1.177	1.966	1.850	1.761	1.771	1.771	1.761
1.152	1.966	1.851	1.759	1.770	1.769	1.760
0.659	1.812	1.659	1.606	1.577	1.615	1.568
0.282	1.632	1.516	1.425	1.428	1.433	1.420

TABLE III.

Measurements of Febr. 7th 1929.

Ice point pressures:

Thermometers I and III: $p_0 = 5.575$ cm Hg" II: $p_0 = 1.135$ " "

Vapour pressure p cm Hg	$T_{I,a}$	$T_{III,a}$	$T_{I,b}$	$T_{III,b}$	$T_{I,c}$	$T_{III,c}$
78.44	4.422	4.336	4.213	4.269	4.255	4.255
35.19	3.663	3.569	3.458	3.502	3.492	3.490
3.568	2.314	2.196	2.104	2.120	2.126	2.113
1.081	1.928	1.811	1.721	1.730	1.739	1.724
0.704	1.807	1.709	1.600	1.626	1.616	1.621
0.619	1.781	1.665	1.576	1.582	1.591	1.577
0.278	1.598	1.499	1.393	1.414	1.407	1.409
0.253	1.589	1.472	1.384	1.385	1.397	1.381
0.201	1.544	1.427	1.339	1.340	1.352	1.335
0.140	1.478	1.387	1.273	1.299	1.285	1.294
0.132	1.470	1.373	1.266	1.284	1.278	1.280
0.0920	1.407	1.315	1.202	1.225	1.214	1.221
0.0595	1.343	1.254	1.141	1.163	1.152	1.159

TABLE IV.

Measurements on May 16th 1929.

Ice point pressures:

Thermometer I: $p_0 = 2.035$ cm Hg" II: $p_0 = 0.06036$ " "

Vapour pressure p cm Hg	T_{Ia}	T_{Ib}	T_{Ic}
76.07	4.603	4.143	4.221
0.220	1.827	1.370	1.395
0.0463	1.536	1.098	1.115
0.0148	1.380	0.959	0.976

In table IV T_{III} fails, as the Wollaston wire in the hot wire manometer was broken.

In table V for the lowest point, the pressure and both temperatures are averages of three independent measurements, closely following each other, so they have particular value.

TABLE V.

Measurements on May 24th 1929.

Ice point pressures:

Thermometer I: $p_0 = 0.1988$ cm Hg" II: $p_0 = 0.02244$ " "" III: $p_0 = 0.3728$ " "

Vapour pressure p cm Hg	$T_{I,a}$	$T_{III,a}$	$T_{I,b}$	$T_{III,b}$	$T_{I,c}$	$T_{III,c}$
76.05	7.49	5.305	4.19	4.14	4.22	4.22
0.0120	2.68	1.765	0.965	0.964	0.970	0.983
0.00465	2.55	1.652	0.897	0.898	0.898	0.897

All pressures are given in cm. of mercury at 0° C. and normal gravity.

The results of the new measurements of the vapour pressure of liquid helium, contained in Comm. N^o. 202*b* and in this communication, can be represented by the two following formulae, in which we have calculated the constants by the method of "least squares", and in which, as the temperature at any given pressure, we have taken the mean of the values given under $T_{I,c}$ and $T_{III,c}$:

for $T > 2.190^\circ$ K.:

$$\log_{10} p_{\text{cm}} = -\frac{3.024}{T} + 2.208 \log_{10} T + 1.217$$

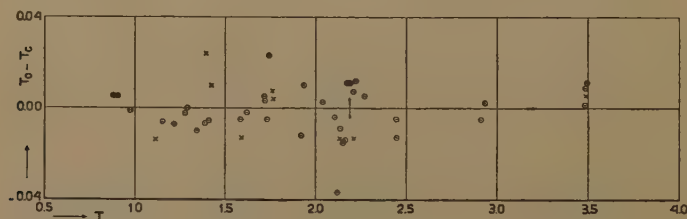
for $T < 2.190^\circ$ K.:

$$\log_{10} p_{\text{cm}} = -\frac{3.859}{T} + 0.922 \log_{10} T + 2.035$$

. . (6)

Here the first formula is drawn through the point $p = 76$, $T = 4.22^\circ$, and the second through the point $p = 3.865$, $T = 2.190^\circ$, which lies on the first curve.

The correspondence between the formulae and the observed results may be judged from Fig. 2, where the abscissa is the temperature, and the difference, $T_0 - T_C$ between the observed and the calculated temperatures is taken as ordinate.



The points, indicated by \odot and \times , have respectively a weight 1 and 0.5.

Fig. 2.

For the moment, we give these formulae only as a purely empirical

representation of the results of these measurements, without going further into questions of detail — in particular the meaning of the behaviour of the curve at $T = 2.19^\circ \text{K.}$, and the way in which the vapour pressure curve will proceed to still lower temperatures ¹⁾. To this we reserve the right to return in due course.

In table VI, we have given, for a number of pressures, the temperatures calculated from the formulae given above. It forms an extension of table IV, given in Comm. N^o. 202*b*.

In comparing with the data given in Comm. N^o. 147*b* for temperatures above the normal boiling point it appears that the first of the formulae (6) gives a sufficiently good correspondence up to 5°K. Above this point the deviations become considerable.

TABLE VI.

Vapour pressure p cm Hg	Temperature T $^\circ\text{K.}$	Vapour pressure p cm Hg	Temperature T $^\circ\text{K.}$
132.9	4.900	0.500	1.538
76.00	4.220	0.250	1.393
50.00	3.795	0.1000	1.236
25.00	3.219	0.0500	1.139
10.00	2.636	0.0250	1.055
5.00	2.298	0.0100	0.960
2.50	2.012	0.0050	0.899
1.000	1.714	0.0025	0.843

¹⁾ Comp. J. E. VERSCHAFFELT, Comm. Leiden Suppl. N^o. 49, p. 23. Nor do we compare them with the results of the measurements of the heat of vaporization.

Physics. — *On the absorption spectra of the azobenzol crystal.* By J. W. OBREIMOW and W. J. DE HAAS. (Communication Phys. Lab. Leiden, N^o. 204c.)

(Communicated at the meeting of November 30, 1929).

Introduction. In two preceding papers ¹⁾ has been shown, that the change of colour of crystals at low temperatures is a general phenomenon for all crystalline bodies and that at low temperatures the absorption spectra of all crystals probably must be line spectra. The wide absorption band of most of the well-known crystals must split up into fine lines like the bands of the rare earths (J. BECQUEREL) or the wide bands of benzol (KÖNIGSBERGER and PRINGSHEIM). The change of colour of the crystals has been attributed to the contraction of the absorption bands.

We remarked that at the temperature of 20° K. the spectrum of the azobenzol is very regular. For want of a suited spectrograph however we could make no accurate measurements.

§ 1. Now Prof. D. S. ROSCHDESTWENSKY in Leningrad has lent us a very good grating (plane grating 8 cm. long, 14000 lines per inch, focal distance of the object-glass ca. 165 cm.). With this spectral apparatus we could obtain spectrograms suited for measurements.

The azobenzol crystallizes from an alcoholic solution in monocline plates ($a : b : c = 2, 11 : 1 : 1, 33$; $\angle (a, c) = 114^{\circ}.26'$). The plane of the plates is c (001). The plane of the optical axis is perpendicular to the plane of symmetry (010). At room temperature in transmitted light the azobenzol-plates are distinctly dichromatic. They have brick colour when the light vector is parallel with the b -axis and lemon colour when the light vector is perpendicular to the b -axis ²⁾).

At 20° K and lower temperatures azobenzol shows a beautiful line spectrum, when the light vector is parallel with the b -axis (red spectrum). We made spectral photographs in the first order of the grating at the temperatures 14° K (liquid hydrogen at 6 cm. pressure) and at 4.22° K. (liquid helium at atmospheric pressure). In the following tables the blackest absorption lines have been given :

¹⁾ J. W. OBREIMOW and W. J. DE HAAS, Proc. Roy. Soc., Amsterdam, **31**, 353, 1929.
J. W. OBREIMOW, Journ. d. Russ. Phys. Ges., 1927.

²⁾ The crystallographic data have been taken from P. GROTH, Chemische Krystallographie

TABLE I.
Azobenzol in liquid helium (4.22°). Red spectrum.

Indication of the line	$\lambda_{air}^1)$	$\nu_{vac} = 1/\lambda_{vac}$	a, b	$\nu_a - \nu_b$	ν_{calc}	$\nu_{calc} - \nu_{vac}$
0	5499.62	18178.03			18178.0	0.0
0'	5440.66	18375.03	0' — 0 =	197.00	18377.3	2.0
0 ₁ '	5427.13	18420.84	0 ₁ ' — 0' =	45.81		
0 ₂ '	5409.66	18480.33	0 ₂ ' — 0' =	105.30		
0''	5382.69	18572.92	0'' — 0' =	197.89	18576.6	3.7
0 ₁ ''	5369.29	18619.39	0 ₁ '' — 0'' =	46.47		
0 ₂ ''	5352.49	18677.64	0 ₂ '' — 0'' =	105.72		
0 ₃ ''	5339.15	18724.37	0 ₃ '' — 0'' =	153.45		
0 ₄ ''	5331.08	18752.72	0 ₄ '' — 0'' =	179.80		
1	5309.13	18830.25	1 — 0 =	652.22	18830.2	0.1
0 ₂ '''	5295.59	18878.40	0 ₂ ''' — 0 ₂ '' =	200.76		
0 ₃ '''	5283.15	18922.85	0 ₃ ''' — 0 ₃ '' =	198.48		
0 ₄ '''	5275.45	18950.47	0 ₄ ''' — 0 ₄ '' =	197.75		
1'	5253.54	19029.50	1' — 1 =	199.25	19029.5	0.0
1 ₁ '	5241.01	19074.99	1 ₁ ' — 1' =	45.44		
1 ₂ '	5225.49	19131.65	1 ₂ ' — 1' =	102.15		
1''	5199.50	19227.28	1'' — 1' =	197.78	19228.8	+ 1.5
1 ₁ ''	5187.38	19272.20	1 ₁ '' — 1'' =	44.92		
1 ₂ ''	5171.05	19333.06	1 ₂ '' — 1'' =	104.78		
1 ₃ ''	5159.48	19376.41	1 ₃ '' — 1'' =	149.13		
1 ₄ ''	5152.63	19402.18	1 ₄ '' — 1'' =	174.90		

In the tables I and II *all* measured lines have been given. Of the 42 measured lines only *two* do not fit into a series schema. This does not mean however, that their right values do not fit into our schema. The signs of interrogation and the remarks in Table II were added during the measurements viz. before the calculations.

All measured lines have been plotted from fig. 1 in wave-number scale.

The spectrum looks as follows; it consists of several series of aequidistant lines:

0; 0'; 0''; 1; 1'; 1''; 1'''; 2; 2'; 2''; 2'''; 3'; 3''.

¹⁾ Last figure not certain.

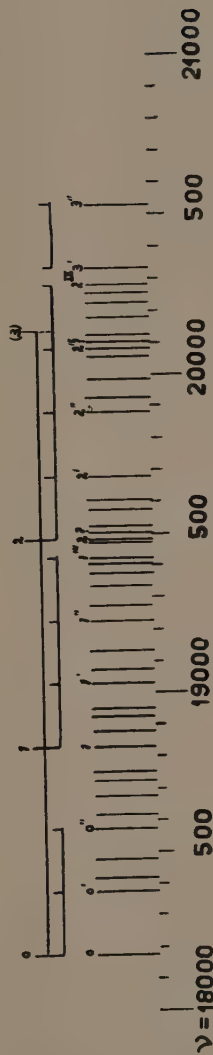


Fig. 1.

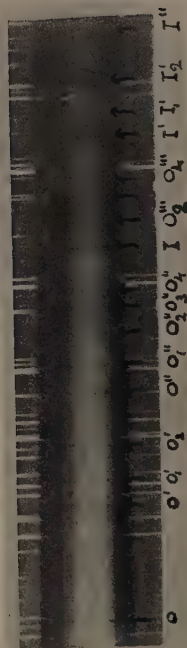


Fig. 3.

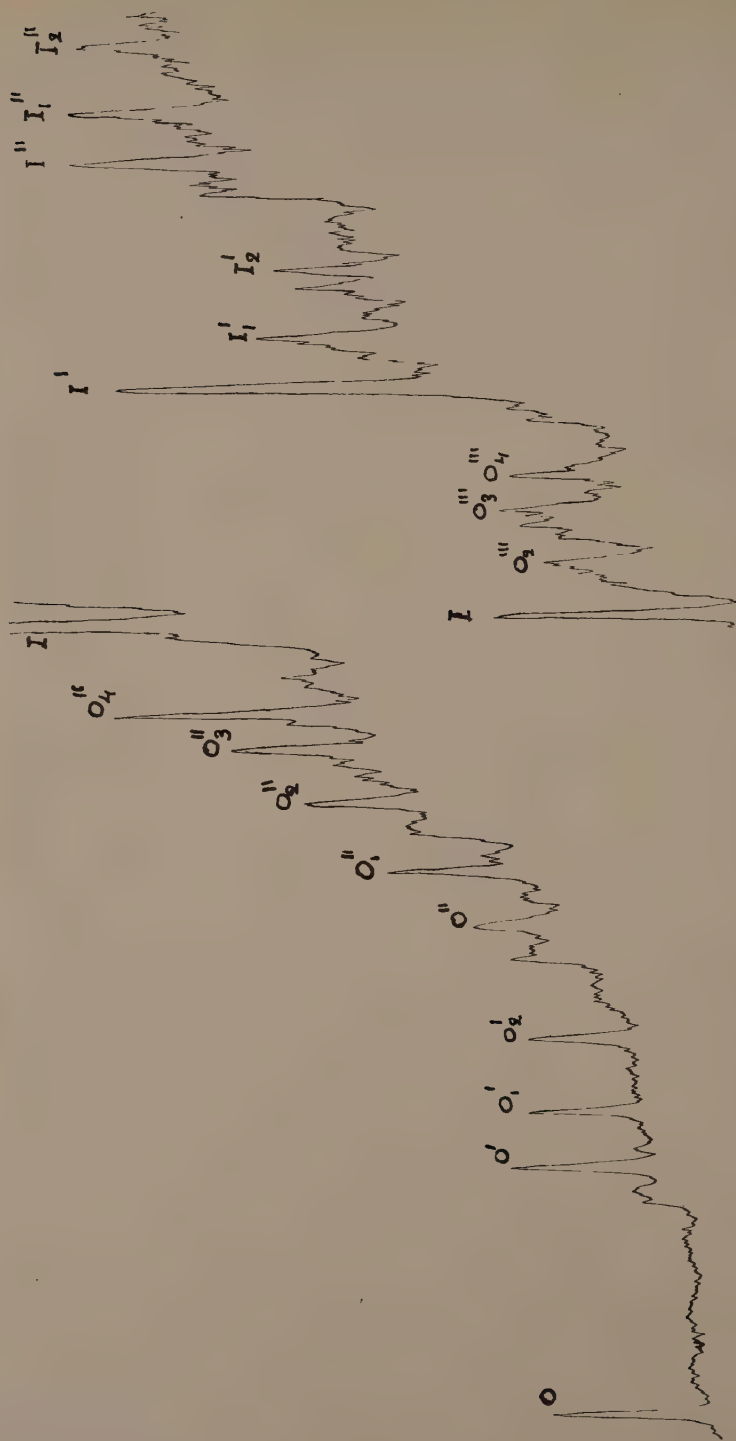


Fig. 2.

where m and n are whole numbers 0, 1, 2, 3, and ν_0 , a , b constants :

$$\left. \begin{array}{l} \nu_0 = 18178.0^3 \\ a = 199.3 \\ b = 652.2 \end{array} \right\} (2)$$

In column 5 of tables I and II we have given the wavenumbers calculated with formula (1) and the constants (2).

But the whole spectrum measured is more complicated. All lines without index, but often also those with the index ' are single, e.g. the lines : 0, 1, 2, 2', 3'.

The next lines (0' ; 1' ; 2'' and as can be seen but not measured 3'') are accompanied by two lines : 0' by the companion lines 0'_1, and 0'_2, 1' by the companion lines 1'_1 and 1'_2. One of the companion lines has a mean distance of 45.0, the other one of 104.0 from the principal line.

The next principal lines are accompanied not only by these two companion lines but by several other companion lines. Among these the satellites 0_3'' ; 1_3'' ; 2_3''' have a mean distance of 147.2 and 0_4'' ; 1_4'' ; 2_4''' of 176.13 from the principal line.

A peculiar aspect of the spectrum is due to the distribution of the intensity in several lines. Let us e.g. consider the group 0.

The line 0 is single or we can say, that the intensity of the lines 0_1, 0_2 is practically zero.

In the reiteration 0' the lines 0' ; 0'_1 ; 0'_2 ; have nearly equal intensities. The lines 0_3' ; 0_4' however are lacking.

These two last lines only occur in the next reiteration 0'' ; here however the intensity of the line 0'' is smaller than the intensity of the line 0_1'' and the line 0_1'' is weaker than 0_2''. In the fourth reiteration however the lines 0''' and 0_1''' are failing ; but the lines 0_2''' ; 0_3''' ; 0_4''' are still present. The same holds for the other groups.

From the microphotogram (fig. 2) of the spectrum of the azobenzol (fig. 3) at 4.22° K. it is evident, that in fact the spectrum is still more complicated. There exist more lines than we have measured. Beside the rather sharp lines we can also see diffuse bands. All these lines and bands constitute a continuous background. This background is weak in the neighbourhood of the line 0 and becomes stronger towards the violet of the spectrum till it is so dark, that no black lines can be discerned against it. That is why in the measurements with the comparator only the lines 0 ; 0' ; 0'_1 ; 0'_2 are discernable. Further towards the violet the lines become less and less distinct. For this reason we could not measure a line beyond 3''. Even in helium we could only extend the measurements till 1_4'', the "helium" crystal being thicker than the "hydrogen" crystal.

Unfortunately we therefore cannot compare the observations in helium and in hydrogen. At 4.22° K. the absorption lines are considerably sharper than at 14° K. It is however remarkable that both at 4.22° K. and at 14° K. the same lines occur with nearly the same distribution of the intensity.

This shows, that our series scheme is also the energy scheme viz. that our fundamental level is also the zero energy level.

From the tables I and II it is evident that at 4.22° K. the position of the lines is shifted with respect to that at 14° K. over 0.2 \AA towards the short wavelengths. The amount at this shifting is uncertain and falls beyond the accuracy of our measurements. The constant a too increases with ν , but our measurements were not accurate enough to give the term with ν^2 .

We also investigated the absorption spectrum of the azobenzol for the component of the light perpendicular to the b -axis, but we could find no structure in this spectrum, not even at 4.22° K. We observed a continuous absorption in the violet without discrete bands.

Azobenzol vaporizes without decomposition. At 70° C. already the azobenzol vapour seems to have a greenish yellow colour. But in the absorption spectrum of the vapour (saturated vapour at 70° C. ; 200° C. and 300° C. in vacuum ; thickness of the layer 10 cm.) we could find no lines or bands. The absorption increased regularly from the red towards the violet.

Physics. — *On the magnetic susceptibility of oxygen as function of temperature and density.* By H. R. WOLTJER, C. W. COPPOOLSE and E. C. WIERSMA. (Communication N^o. 201d from the Physical Laboratory at Leiden). (Communicated by Prof. W. J. DE HAAS).

(Communicated at the meeting of November 30, 1929).

§ 1. *Introduction.* The magnetic susceptibility of a paramagnetic substance (χ) is a function of temperature (T) and density (ϱ). Usually the experimental variations in the latter are of no importance. For solid substances the density, or at least the paramagnetic density, has been varied artificially by studying salts with more or less crystall-water.¹⁾ Much simpler, in nearly every respect, is the case of a liquid or a gas. PERRIER and KAMERLINGH ONNES²⁾ have investigated the susceptibility of liquid mixtures of oxygen and nitrogen (the latter being considered merely as a magnetically neutral diluting agent). They found that their results agreed very well with the formula:

$$\chi (T + \Delta) = \text{constant} = 0.0315_2$$

in which Δ is a function of the density only, that becomes zero in the limiting case of a density zero. This formula has a very simple meaning. As the authors point out themselves: "the change in the density of oxygen only alters the specific magnetisation without changing the CURIE constant". $1/\chi$ is a linear function of T and the density function Δ indicates the existence of a negative molecular field, the dependence of which on the density or the distance of the molecules can be derived from the experiments. Moreover we may point out the remarkable fact that extrapolation of this formula to normal density and atmospheric temperature furnished a value for the susceptibility of gaseous oxygen at 20° C. and 1 atm. only moderately agreeing (difference 2^o/₀) with the value then known, but perfectly with the value found afterwards by BAUER, WEISS and PICCARD³⁾ (difference 1^o/₀₀).

1) L. C. JACKSON, Leiden Comm. N^o. 163.

2) A. PERRIER and H. KAMERLINGH ONNES, Leiden Comm. N^o. 139.

3) H. KAMERLINGH ONNES, Leiden Comm. Suppl. N^o. 44a I.

The experiments referred to a considerable range of densities ($\rho = 1.204$ to 0.080 gram/cm³), but of course to a limited range of temperatures only ($77^{\circ}.4$ K. $-64^{\circ}.2$ K.).

Then the question arose whether this formula would hold over a large range of temperatures and for oxygen in the gaseous state. KAMERLINGH ONNES and OOSTERHUIS¹⁾ had investigated previous to the experiments just discussed the susceptibility of oxygen of more than 100 times the normal density and they concluded that down to -130° C. no deviation from CURIE's law of inverse proportionality to the absolute temperature could be found. According to the experiments of PERRIER and KAMERLINGH ONNES, however, to this density $\Delta = 5.4$ would correspond and later inspection of the results about oxygen at 100 times the normal density showed that the formula $\chi(T + 5.4) = \text{const.}$ satisfied the results even slightly better than the strict CURIE formula, though much depends on the weight attached to the lowest temperature values of the series.

So it seemed interesting to investigate the susceptibility of gaseous oxygen at high densities in dependence on temperature. In the following the results of 5 series relating to 3 different densities are given.

§ 2. *The experiments.* A thick walled bronze tube is filled with oxygen at the required density. By means of a german silver capillary it is connected to a reservoir containing pure oxygen. In some of the series this oxygen was prepared from potassiumpermanganate, condensed by means of liquid air and distilled in the reservoir, in other series we started from commercial oxygen and distilled it. By cooling the bronze tube in liquid air the oxygen condensed into it from the reservoir until a previously arranged pressure was reached, then a cock between the tube and the reservoir was closed, the german silver capillary was pinched tightly at its top, cut off from the rest and soldered. Sometimes a small brass cap was soldered over it. Then the tube was taken from the liquid air, allowed to come at atmospheric temperature, weighed and put into the magnetic apparatus. In the first series this was the apparatus used by PERRIER and OOSTERHUIS²⁾, in the other series the horizontal vapour cryostat³⁾; in the first series the susceptibility was determined at different field intensities and absolute measurements were taken, in the other series values relative to that at a certain temperature were determined always in the same field, density was determined from the weight of the full and the empty tube. After each series the magnetic attraction on the empty tube has been carefully studied.

Usually the temperature was not more lowered than to a few degrees above the critical point of oxygen (154° K.).

¹⁾ H. KAMERLINGH ONNES and E. OOSTERHUIS, Leiden Comm. N^o. 134d.

²⁾ H. KAMERLINGH ONNES and A. PERRIER, Leiden Comm. N^o. 139a.
E. OOSTERHUIS, Leiden, Comm. N^o. 139b.

³⁾ E. C. WIERSMA and H. R. WOLTJER, Leiden Comm. N^o. 201c.

§ 3. Results.

1st Series $\varrho = 0.428$				4th Series $\varrho = 0.320$			
T	$10^6 \chi$	$\chi_{291} : \chi$	$\frac{\chi_{291}}{\chi} - \frac{T}{291}$	T	$10^6 \chi$	$\chi_{291} : \chi$	$\frac{\chi_{291}}{\chi} - \frac{T}{291}$
284.97 K.	100.1	0.9766	+ 0.003	292.3 K.		1.006	+ 0.001
250.46	113.4	0.861	+ 0.001	289.9		0.993	- 0.003
236.66	120.9	0.809	- 0.004	255.5		0.877	- 0.001
224.67	126.5	0.773	+ 0.001	231.6		0.788	+ 0.002
213.00	133.8	0.731	- 0.001	212.1		0.726	- 0.003
197.99	144.7	0.676	- 0.005	210.9		0.721	- 0.003
170.38	165.2	0.592	+ 0.007	191.7		0.657	- 0.001
158.91	175.1	0.559	+ 0.013	166.3		0.573	+ 0.002
2nd and 3rd Series $\varrho = 0.443$				157.3		0.548	+ 0.008
				5th Series $\varrho = 0.152$			
288.5		0.992	0.000	289.7		0.997	0.000
288.2		0.986	- 0.004	286.8		0.988	+ 0.002
286.6		0.985	0.000	287.0		0.996	- 0.001
285.9		0.983	0.000	285.9		0.988	+ 0.005
285.5		0.980	- 0.001	247.0		0.845	- 0.004
283.9		0.969	- 0.006	226.2		0.780	+ 0.002
260.9		0.895	- 0.002	206.0		0.708	+ 0.001
257.5		0.880	- 0.005	185.4		0.639	+ 0.002
233.8		0.800	- 0.003	177.2		0.613	+ 0.004
231.5		0.792	- 0.004	155.3		0.542	+ 0.008
228.9		0.786	- 0.001				
215.8		0.739	- 0.003				
211.2		0.725	- 0.001				
207.2		0.710	- 0.002				
199.8		0.688	+ 0.002				
180.9		0.623	+ 0.002				
171.8		0.592	+ 0.002				
158.4		0.553	+ 0.009				
156.9		0.548	+ 0.009				

The values of χ at 291° K. for $\varrho = 0.443$, 0.320 and 0.152 are found to be 1207 K., 1237 K. and 1288 K., where K is a constant of the apparatus. If one extrapolates these values to $\varrho = 0$ and takes for $\chi_{f=0, 291^\circ}$ K, the value of BAUER, WEISS and PICCARD, one finds

$\varrho = 0.443$	0.320	0.152
$10^6 \chi_{291^\circ} = 98.2$	100.7	104.9

In fig. 1 $\frac{\chi_{291}}{\chi} - \frac{T}{291}$ has been plotted against T , thus showing the deviations from CURIE's law. In the same graph by straight lines the devia-

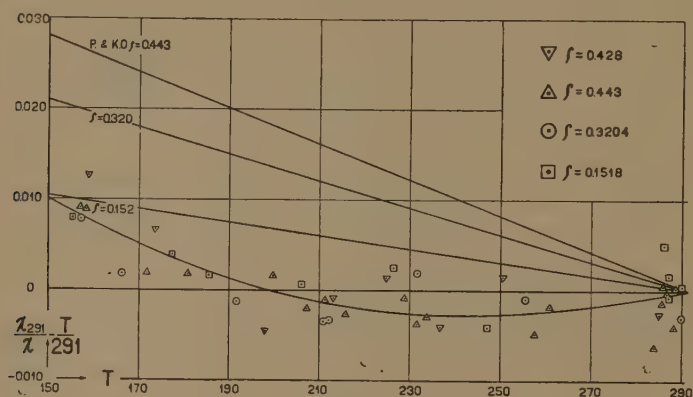


Fig. 1.

tions are represented that would exist, if the formula $\chi(T + \Delta) = \text{constant}$ was satisfied with values of Δ deduced from the work of PERRIER and KAMERLINGH ONNES. According to them (for liquid oxygen) the following values of ϱ and Δ would correspond:

ϱ	0.428	0.443	0.320	0.152
Δ	17.5	18.0	13.2	6.4

§ 4. *Conclusions.* From the numerical data and from the graph there appear deviations from CURIE's law. However, the character of these deviations is absolutely different from that included in the formula $\chi(T + \Delta) = \text{const.}$ with which our results in no way can be reconciled. The principal features of the deviations shown by our results are the following:

1. Down to about 175° K. the deviations from CURIE's law are within the limits of accuracy (about $5^\circ_{/00}$) but below that temperature they become marked. The whole of the observations show a decidedly curved

line for $\frac{\chi_{291}}{\chi} - \frac{T}{291}$ as function of T .

2. There is no systematic difference for the different densities.

One would be inclined to represent the observations by the formula $(\chi - \chi_0) T = \text{const.}$, χ_0 being a function of the density. Then χ_0 must satisfy the condition, on account of the results of PERRIER and KAMERLINGH ONNES, that it becomes zero for a limit density zero. On the other hand it must be such, according to our experiments, that $\frac{\chi}{\chi_{291}} - \frac{T}{291}$ is (nearly) independent of the density. Both conditions are not to be reconciled.

If the small deviations of CURIE's law between 291° K. and 175° K. are neglected, the observations may be represented by $\chi T = \text{constant}$, but now the value of the constant depends on the density,

viz.	$\rho = 0.443$	0.320	0.152	0.00
	$10^2 C = 2.86$	2.93	3.05	3.15.

This implies a dependency of the atomic moment of oxygen on the density.

In comparing these results with the theoretical investigations by J. H. VAN VLECK¹⁾ we see that there is no difficulty to explain the fact that there is no dependence of Δ on density, however for the dependence of the CURIE-constant from density we must think of intermolecular fields or association as he states that: "The susceptibility per molecule should not change with pressure except in so far as molecular association, intermolecular fields, etc. are involved, as they often are to a great extent in liquids."²⁾

§ 5. *Summary.* The specific magnetic susceptibility of gaseous oxygen at temperatures between 291° and 156° K. and densities from 0.443–0.152 cannot be represented by the formula $\chi(T + \Delta) = \text{const.}$ in which Δ depends on the density as in the experiments of PERRIER and KAMERLINGH ONNES for liquid oxygen at different concentrations.

From 291° to about 175° K. CURIE's law is nearly followed with a CURIE-constant (and so an atomic moment) dependent on the density. There is however a slight but systematic deviation in the sense of a curvature of the $1/\chi$, T -lines. This deviation becomes marked below 175° K.

In conclusion we have pleasure in thanking Prof. Dr. W. J. DE HAAS very much for affording us the opportunity for this research.

¹⁾ J. H. VAN VLECK, Phys. Rev. **29**, 727 (1927); Id. **30**, 31 (1927); Id. **31**, 587 (1928)

²⁾ L. c. **30**, 38.

Physics. — *Intensities in the He-Spectrum as a function of the pressure and the electron velocity.* By W. ELENBAAS. (Communicated by Prof. L. S. ORNSTEIN.) (Communication from the Physical Institute of the University of Utrecht.)

(Communicated at the meeting of November 30, 1929).

At a pressure of 0.1 mm. and 0.04 mm. the excitation-functions of the visible lines are measured. All lines except those of the Triplet principal and sharp subsidiary series, which have a sharp maximum, show two maxima. Lines of one series have similar excitation curves. The ratio of the intensities at pressures of 0.1 mm. and 0.04 mm. waxes with the electron velocity, particularly so for lines of the diffuse subsidiary series. The triplet-singlet ratio increases with decreasing electron velocity, especially the

$$\text{ratio } \frac{2^3S-3^3P}{2^3S-3^1P}.$$

The decrement in the series is not very dependent on the electron velocity but does however depend very much on the pressure. The triplet-singlet ratio shows slight increase with the pressure. The excitation function of the sparkline $\lambda = 4686 \text{ \AA}$ is measured. This line is not found below 75 Volts.

The polarisation of the arclines and the sparkline is investigated as a function of the electron velocity. The degree of polarisation $P = 100 \frac{I_{\parallel} - I_{\perp}}{I_{\parallel} + I_{\perp}}$ is zero at the excitation potential and has a maximum at approximately double this voltage.

§ 1. In a former publication ¹⁾ we showed that in the excitation of the helium by means of electron impact the following laws are obeyed.

10. As the pressure is lowered the singlet system is more pronounced.

20. Lowering of the pressure causes the higher lines of the series to become more pronounced.

30. Decreasing electron velocity is accompanied by a more pronounced triplet system.

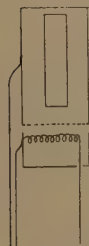


Fig. 1.

Further the excitation functions for several lines were determined. Now we shall communicate several measurements obtained with other tubes. The advantage of these tubes is that the spiral is also enveloped by a cylinder to prevent electrons from striking the cylindrical anode on the outside (fig. 1).

Secondly a thin metallic wire was wrapped round the anode to prevent, as

¹⁾ M. G. PETERI und W. ELENBAAS. ZS. f. Phys. 54. 92, 1929.

far as possible, even the observation slit from acting as a gap in the electrical screen. The electrons pass through the grid into a fieldfree space where they excite the He . The decrease in intensity at greater distances from the grid will be described in a further publication.

In this work the portion of the observation slit which is nearest to the grid is projected on to the slit of the spectrograph. Photographs are obtained at varying potential between grid and spiral, but constant grid current and constant exposure (the spiral current naturally varying accordingly).

In this way about twenty spectra are photographed on one plate.

In order to convert densities into intensities the method of varying slitwidth in exposures with a source emitting a continuous spectrum was used.

In order to compare the intensities of the different lines the following method was adopted: A lamp with a calibrated energy-wavelength curve was photographed at three different distances with various slitwidth for each distance. At the smallest of these distances the densities in the violet were sufficient and at the greatest distance this was the case in the red. Between these two extremes a position can be found such that the density is convenient at 4900 to 4000 \AA . In two consecutive positions of the lamp the densities were sufficient in a mutual range; this renders the possibility of obtaining the conversion factor for intensities with regard to one of the two positions. This factor was found to be approximately equal to the quotient of the square of the distances. The advantage of the use of this method is that the abovementioned factor is independent of the wavelength, which is not the case when reducers are used.

§ 2. *The Excitation function.*

The intensities for the different lines obtained in this way at a pressure of 0.1 mm. are given in logarithmic scale in fig. 2 as a function of the potential. This gives a good survey of the way in which the intensity ratios alter with electron velocities. At high potentials the green line 5016 (2^1S-3^1P) is the strongest line in the visible region. At about 36 Volt the line 3889 ($3S-3^3P$) becomes the strongest.

As the logarithmic scale distorts the curves considerably one curve in each series is drawn in figs 3 and 4. Fig. 3 represents singlet lines and fig. 4 represents triplet lines. Several of these lines are multiplied by a factor to give nearly equal maxima. All lines except those of the series 2^1S-m^1P have a maximum between 30 and 40 Volts. A second blunter maximum is obtained at higher voltages except at lines of the series 2^3P-m^3S and 2^3S-m^3P .

HANLE¹⁾ finds one maximum in each curve and this maximum corresponds with our first maximum in several cases (2^1P-m^1S and

¹⁾ W. HANLE. ZS. f. Phys. 56. 94. 1939.

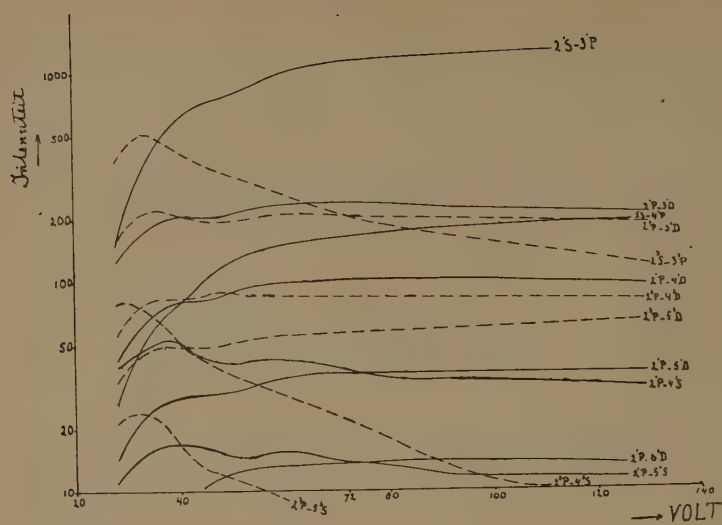


Fig. 2.

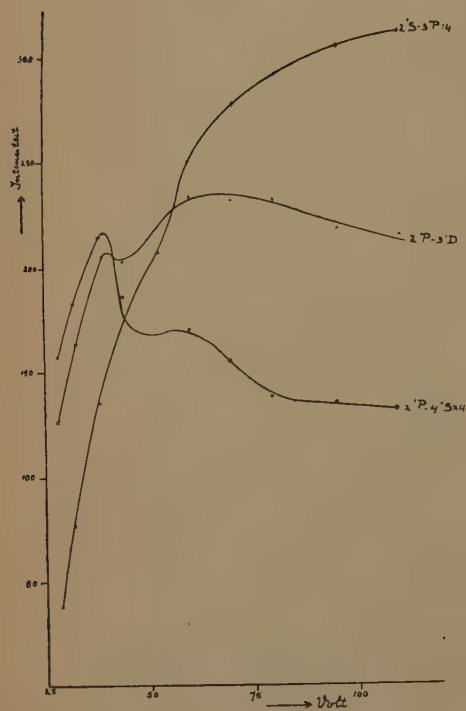


Fig. 3.

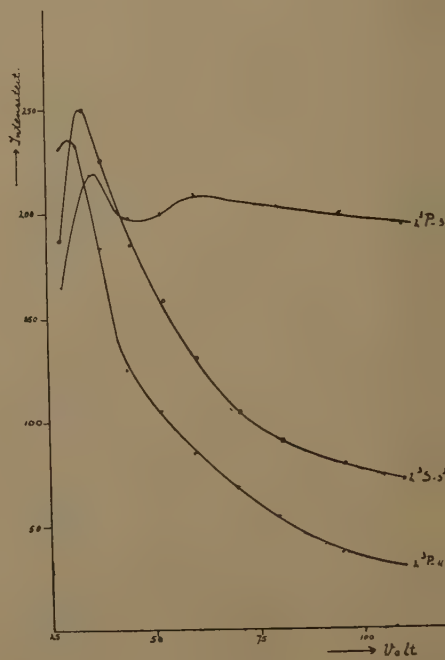


Fig. 4.

2^3P-m^3D) while in other cases it corresponds with our second maximum (2^1P-m^1D and 2^1S-m^1P). The excitation curves of the series 2^3P-m^3S and 2^3S-3^3P correspond with those of HANLE. Here we also found only one maximum and this is also the most pronounced.

One should suggest that the second maximum is a repetition of the first; for instance the line 2^1P-4^1S ($\lambda = 5048 \text{ \AA}$) has a blunt maximum at 60 Volts. But now, among these 60 Volts electrons will also be present those which have already excited once thus retaining a velocity of 35 to 40 Volts, enabling them to give a maximum. The fact that the 60 Volts maximum is blunter is explained by the fact that only a portion of the electrons have a velocity of 35 to 40 Volts. This explanation cannot however be correct. Firstly the relative number of electrons which have already lost 20 to 25 Volts is small directly behind the grid and secondly even if this number were sufficient to produce the effect then surely the lines which have the sharpest first maximum should also have the sharpest second maximum whereas these lines, namely 2^3P-m^3S and 2^3S-m^3P are the very lines which have no second maximum.

As therefore the second maximum appears in definite series and as it has also been found by HANLE in several cases (HANLE did not in this case find the first maximum) we must accept that both maxima are characteristic for the excitation curves.

§ 3. Dependence of excitation curves on the pressure.

The excitation curves were also determined at 0.04 mm. Here the form is the same but the fall in intensity after the maximum is steeper than in the case of 0.1 mm. For each line the ratio of the intensities at 0.1 mm. and 0.04 mm. varies with the electron velocity. This ratio is traced (fig. 5)

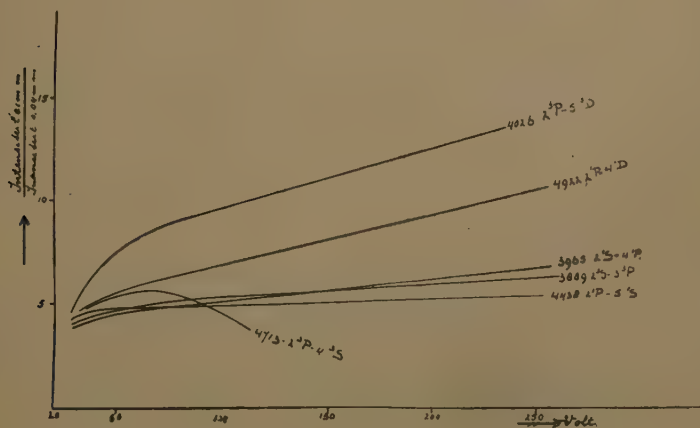


Fig. 5.

as a function of the voltage for one line in each series. On the assumption that there is no interaction between the atoms one should expect the curves (fig. 5) to reduce to one straight line parallel to the voltage abscissa at an ordinate of $2\frac{1}{2}$. From fig. 5 we see that the deviation from a straight line increases with increasing electron velocity, and that the greatest deviation takes place in lines of the diffuse subsidiary series. This effect is explicable if one assumes that an excited atom passes to another state on collision with an atom.

Since this excitation by collisions increases with increasing voltage the former must take place from a level that is of very frequent occurrence at great electron velocities. The 1P levels fulfil this condition, because the excitation curves of the lines 5016 \AA° and 3965 \AA° rise very rapidly with increasing electron velocity and also the absolute strength of these lines is great.

We therefore assume that 1D levels can arise from P levels by collision with atoms. The energy difference can be converted into kinetic energy of the atoms.

At high pressures the BOLZMANN distribution is established as a result of these collisions. Further if we accept that the 3D levels can arise from 1D levels then the behaviour of the lines of the diffuse subsidiary series is explained; at high electron velocities, the excitation by collisions is predominant over direct excitation due to the excess of 1P levels and small direct excitation. The 1D levels obtain a concentration proportional to the square of the pressure and the 3D levels obtain a concentration proportional to the third power of the pressure. As is seen in fig. 5 this is more or less the case.

At the S and P levels these excitation by collisions evidently play a smaller part.

§ 4. Triplet singlet ratios.

In fig. 6 the triplet-singlet ratio is plotted in logarithmic scale as a function of the electron velocity at a pressure of 0.1 mm. The ratio $\frac{2^3S-3^3P}{2'S-3'P}$ shows the greatest dependence. From 28 to 110 Volts the value of $\frac{2^3S-3^3P}{2'S-3'P} \left(\frac{3889}{5016} \right)^4$ falls from 0.915 to 0.0407.

All curves rise with decreasing voltage. At 0.04 mm. we find similar curves lying somewhat lower.

§ 5. Excitation Curve of the sparkline $\lambda = 4686 \text{ \AA}^\circ$.

The intensity of the sparkline is measured as a function of the electron velocity at a pressure of 0.1 mm. In fig. 7 the results are plotted. The unit of intensity is the same for the arc- and sparklines.

To excite the begin level of the sparkline one requires 75.3 Volts.

Double ionisation can take place at 78.7 Volts. It is not clear whether the curve reaches the voltage abscissa at 75.3 Volts or 78.7 Volts. The

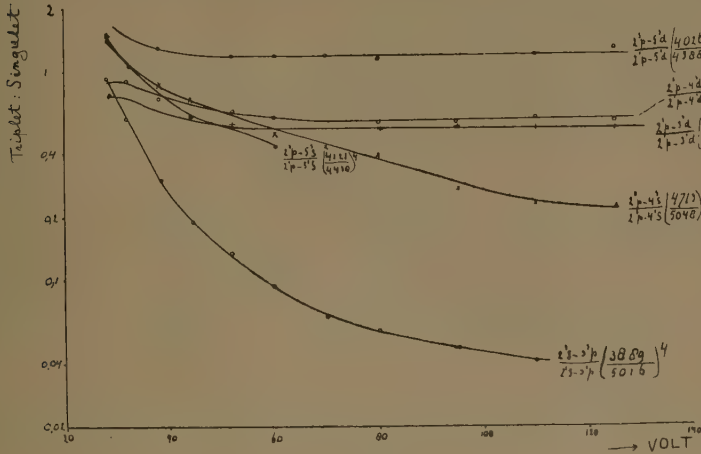


Fig. 6.

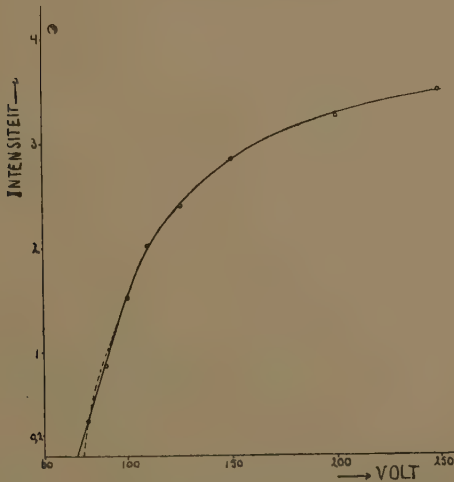


Fig. 7.

intensity of the sparkline is exceedingly weak below 75.3 Volts. The probability that this excitation takes place in two successive processes appears to be very small, because otherwise the sparkline would appear above 50 Volts. The He atom could then first be ionised and then the sparkline level could be produced by a second electron.

There are two possible explanations of the missing sparkline below 75 Volts :

10. The concentration of the *He* ions is very small below 75 Volts.

20. The excitation probability of an ion by a second electron is small.

As we have no reason to accept the latter we must accept that the ionconcentration is small and that therefore in the formation of arc lines the recombination spectrum plays a minor part.'

§ 6. *Comparison with Geisslertubes.*

At 28 Volt and 1 m.A. the 0.1 mm. tube was compared with two Geisslertubes, one having a pressure of 6.9 mm. and a capillary crossection of 2.18 mm²., the other a pressure of 12 mm. and a crossection of 6 mm² The current density was chosen equal in the three tubes. This gave a current of 0.047 mA and 0.129 mA resp. in the Geisslertubes. To obtain this, about 1000 Volts constant voltage had to be applied to the ends of the Geisslertubes.

The Geisslertubes were compared with 0.1 mm. tube at the smallest voltage, because the electron velocity in the Geisslertubes is probably small. The mean free path of electrons in *He* at a pressure of 6.9 mm. is approximately 0.01 cm. Hence the increase in velocity of the electron between two collisions will be of the order of 1 Volt. In order then, to increase the velocity from 20 to say 30 Volts it must collide elastically many times. In generally however before gathering so much energy, it will collide inelastically. This leads to the result that the average energy of the exciting electrons perhaps lies below 28 Volts.

The results are tabulated (table I). The factors of the columns *d* and *e* become smaller for higher lines of the series. This therefore means that the first lines of the series are favored at higher pressures. As can be seen from fig. 2, where the distance of the curves of lines of one series is hardly dependant upon the electron velocity, the decrement in the series is hardly dependant upon electron velocity.

The ratio of the intensities at 6.9 mm. and 0.1 mm. (column *d*) is smaller than 69 for all lines. At the lower pressures 0.04 and 0.1 mm. the intensity rose more rapidly than the pressure, also at small electron velocities. That this was not so in the abovementioned case must probably be accounted for by greater absorption. That the intensities at 12 mm. are so small must be attributed further to the greater capillary crossection, because also the latter causes greater absorption. A third cause of this small intensity can be the smaller electronvelocity in the 12 mm. tube.

The triplet singlet ratios at 12 mm., 6.9 mm., 0.1 mm. 28 Volts, and 0.04 mm. 28 Volts are tabulated (table II).

At 6.9 mm. the triplet singlet ratio is somewhat greater than at 0.1 mm. 28 Volts. The fact that the ratio at 12 mm. is greater than at 6.9 mm. must, in addition to the greater pressure, be attributed to the smaller electron velocity .

TABLE I.

A°		a	b	c	d	e
		12 mm	6.9 mm	0.1 mm 28 Volt	b/c	a/b
6678	2 ¹ P—3 ¹ D	777	6550	125	52.3	0.118
4922	2 ¹ P—4 ¹ D	38.7	460	42	11.2	0.084
4388	2 ¹ P—5 ¹ D	7.1	88	14	6.2	0.080
4144	2 ¹ P—6 ¹ D	2.08	33.8			0.061
5048	2 ¹ P—4 ¹ S	16.5	285	39.5	7.2	0.058
4438	2 ¹ P—5 ¹ S	2.6	42.4	11	3.85	0.061
5016	2 ¹ S—3 ¹ P	162.5	2550	147	17.4	0.064
3965	2 ¹ S—4 ¹ P	7.6	131	26	5	0.058
5876	2 ³ P—3 ³ D	1540	7200	157	46	0.214
4472	2 ³ P—4 ³ D	171	1790	55	32.5	0.096
4026	2 ³ P—5 ³ D	22.5	300	33	9.1	0.075
4713	2 ³ P—4 ³ S	55.5	575	77	7.45	0.096
4121	2 ³ P—5 ³ S	5.5	83	21	3.95	0.066
3889	2 ³ S—3 ³ P	990	8500	374	22.8	0.116

TABLE II.

	12 mm	6.9 mm	0.1 mm 28 V.	0.04 mm 28 V.
$\frac{2^3S-3^3P}{2^1S-3^1P} \left(\frac{3889}{5016} \right)^4$	2.2	1.2	0.915	0.85
$\frac{2^3P-3^3D}{2^1P-3^1D} \left(\frac{5876}{6678} \right)^4$	1.17	0.66	0.75	0.69
$\frac{2^3P-4^3D}{2^1P-4^1D} \left(\frac{4472}{4922} \right)^4$	3.05	2.67	0.90	0.805
$\frac{2^3P-5^3D}{2^1P-5^1D} \left(\frac{4026}{4388} \right)^4$	2.28	2.45	1.70	1.41
$\frac{2^3P-4^3S}{2^1P-4^1S} \left(\frac{4713}{5048} \right)^4$	2.55	1.53	1.48	1.21
$\frac{2^3P-5^3S}{2^1P-5^1S} \left(\frac{4121}{4438} \right)^4$	1.57	1.46	1.42	1.19

We thus say: the fall in the series is chiefly determined by the pressure and the triplet singlet ratio by the electron velocity and also though to a smaller degree, by the pressure. The yellow lighting of the *He* Geisslertube is probably also caused by the small electron velocity. From figure 2 can be seen that below 28 Volts the yellow line $\lambda = 5876 \text{ \AA}^\circ$ becomes the most intense of the visible lines, because the line 3889 \AA° is not in the visible region.

§ 7. Polarisation of the light.

According to SKINNER¹⁾ and SKINNER and APPLEYARD²⁾ the light produced by a parallel electronstream in mercury is partly polarised and in most cases the electric vector is parallel to the electron stream. They found that the degree of polarisation defined as $P = 100 \frac{I_{||} - I_{\perp}}{I_{||} + I_{\perp}}$ was zero at the excitation potential, then increased to a maximum and decreased even to negative values at high velocities. As there were no experiments in the case of *He*, we measured the degree of polarisation as a function of electron velocity. The observation-slit was again projected on the slit of the spectrograph. A Nicol prism was placed between the lightsource and achromatic lens (close to the lightsource). The aperture of the beam was so small that neither in the Nicol, nor in the spectrograph, was there any light cut off from the beam. Since the image of the source moved slightly as the Nicol was turned, the projection was not sharp. Through the use of narrow beams, Nicol and unsharp images the necessary exposure would have been very long and for this reason the measurements were carried out at stronger currents, namely 3 mA. (this grid current was reached at 28 Volts and maximum glow of the spiral). In this way we obtained exposures of 12 to 24 hours, at a pressure of 0.04 mm. To be free from polarisation in the spectrograph, a continuous source was also photographed on each plate in both positions of the Nicol. In the one position the intensity was 10 % stronger than in the other. This amount therefore had to be subtracted from the intensity of the line when the Nicol was in the former position.

In figs. 8 and 9 the degree of polarisation is plotted as a function of the electron velocity for several lines at 0.1 mm. and 0.04 mm. resp. In all cases P was positive. The form of the curves agrees with those found by SKINNER and APPLEYARD for *Hg*.

In their work the maximum also lies at approximately double the excitation voltage.

That the change of sign of P was not observed in the case of *He* is probably due to the fact that we did not work at sufficiently high velocities.

¹⁾ H. W. B. SKINNER. *Proc. Roy. Soc.* **112**. 642. 1926.

²⁾ H. W. B. SKINNER and E. T. S. APPLEYARD. *Proc. Roy. Soc.* **117**. 224. 1927.

In Hg the change of sign occurs at approximately nine times the excitation potential. This corresponds to a potential of approximately 200 Volts in

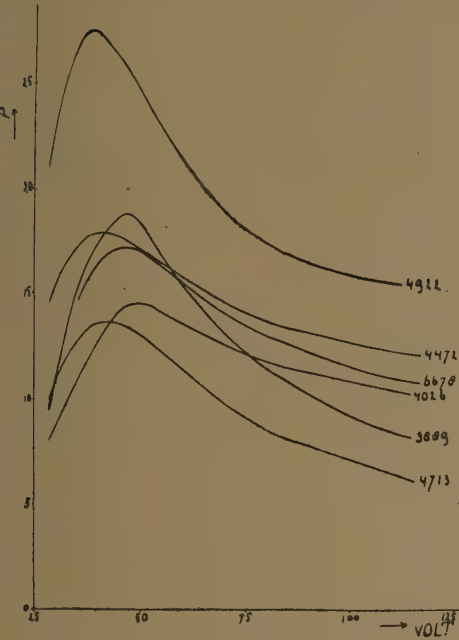


Fig. 8.

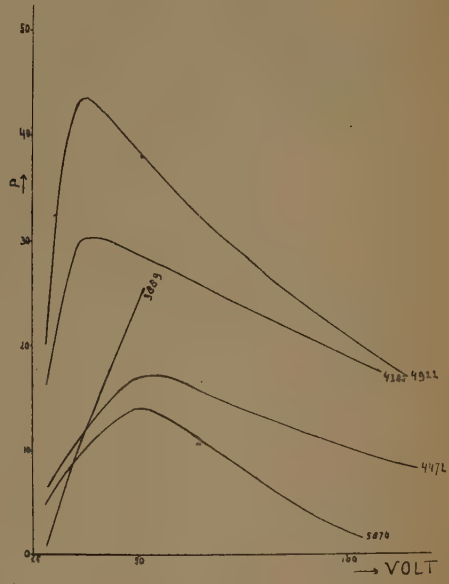


Fig. 9.

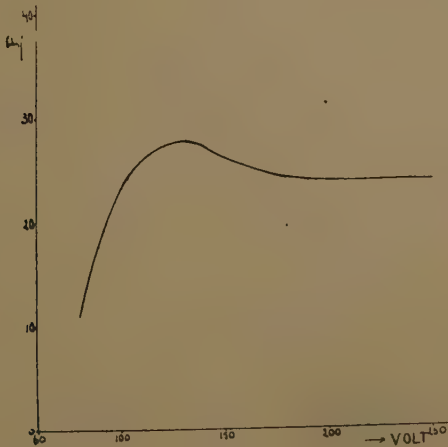


Fig. 10.

the case of *He*. In an attempt to measure the degree of polarisation at this potential the spiral fused.

Also the degree of polarisation of the sparkline was measured as a function of the electron velocity. Fig. 10 gives the result. There, too the degree of polarisation tends to zero at the excitation potential.

May I take this opportunity of thanking Prof. L. S. ORNSTEIN for stimulating me in this work and for his constant interest.

My thanks are also due to Mr. A. M. VAN DOMMELEN for his assistance in the photometry of the plates.

Physics. — *Optical determination of the sphere of action of the He atoms for electrons.* By L. S. ORNSTEIN and W. ELENBAAS. Communication from the Physical Institute of the University of Utrecht.)

(Communicated at the meeting of November 30, 1929.)

The sphere of action of the *He* atom is determined from the reduction in intensity, of the heliumlines with increasing distance from a grid in a field-free space.

§ 1. *Method.*

By the measurement of the excitation function of *He* in a field-free space of cylindrical form, we observed that the light intensity decreased with increasing distance from the grid through which the electrons passed into the field-free space. It appears that this reduction originates in collisions of electrons with *He* atoms, whereby the electrons are deflected from their paths or suffer a reduction in speed. If this is the case then the mean free path of the electrons in *He* gas can be determined by measuring the intensities of the lines at different distances from the grid.

The arrangement which was used here was the same as that used in the measurement of the excitation function, with the difference, that now the observation slit of the cylinder was set vertical, whereby it was possible to project an image of the total length of this slit on the slit of the spectrograph with an achromatic lens.

In order to reduce the passage of the lines of force through the observation slit, the anode was wrapped with thin wire in front of the slit, the windings being 1 or 2 mm. apart. These windings made it much easier to obtain a sharp image of the observation slit on the slit of the spectrograph. In this manner we obtain long lines on the plate, and these lines were divided by the image of the windings in several places. Every line was photometred in several of the spaces between two windings. These windings made it easier to identify the same place on every line. In order to calculate the relation between density and intensity the method of variation of the slit width was used, with a lamp giving a continuous spectrum.

§ 2. *Measurements with a cylinder of 15 mm. length and He pressure 0.1 mm.*

If we call the intensity of the light 100 at the beginning of the space (where the electrons enter), then we find for an electron velocity of 30 and 76 Volt, respectively, the values given in Table I for distances of 2.47, 5.45, 9.05, 11.27 and 13.5 mm.

TABLE I.

30 Volt							76 Volt					
$\begin{array}{c} x \\ \backslash \\ A^\circ \end{array}$	0	2.47	5.45	9.05	11.27	13.5	0	2.47	5.45	9.05	11.27	13.5
5876	100	68	43	26.5	21.5	17	100	76.5	50.5	35.5	31.5	27.5
5016	100	66	41.5	30	24.2	19.1	100	73	51	35.5	30	25
4922	100	73.5	47	31	26.6	21.5	100	81.5	60	42.5	35	30.5
4713	100	71	42.8	35	29.8	24.8	100	80	61.5	46.5	46	45
4472	100	71.5	43	31.5	24.5	19.2	100	68.5	43.5	32.5	26	21.2
4438	100	70	43.8	29.7	24.4	20.6	100	70.5	46.5	36.5	31.5	29.4
4388	100	66	40.5	29	24.2	19	100	65	42.5	32	25	20.3
4026	100	63	40	28.2	21.5	17.6	100	71.5	49	35.5	29	24.5
3965	100	73.5	43	29.5	24.3	19	100	67.5	44	31	25.5	20.5
3889	100	73	48	33.2	26	21	100	74	52	39.5	33.7	28.5
Mean	100	69.7	43.3	30.4	24.7	20.8	100	72.8	50.1	36.7	31.3	27.3

In the absence of any subsidiary phenomena, we can expect that the intensity along the observation slit falls off as $e^{-\frac{x}{\lambda}}$, where x is the coordinate in the direction along the slit, and λ is the mean free path of the electrons in helium.

If we plot the mean value of the intensity of the lines on a logarithmic scale against the distance from the grid, we expect to obtain a straight line, from the inclination of which λ can be calculated. In fig. 1 this is shown for 30 Volts. The first part of this line is straight, but at greater distances from the grid the measured intensity becomes greater than that of the extrapolated straight line; the measured points are shown thus 0. In order to explain this we tried to apply the theory of back diffusion as it has been developed for the passage of electrons through metal foil. From this we find for the electron density at a distance x from the grid:

$$E = \frac{E_0(1+p)}{1-p^2e^{-2\alpha\xi}} (1 - pe^{-2\alpha(\xi-x)}) e^{-\alpha x} \quad 1)$$

p = constant of back diffusion.

α = absorption coefficient

ξ = total length.

For the part which depends on x we may write:

$$e^{-\alpha x} - ke^{\alpha x} \text{ (where } k \text{ is } pe^{-2\alpha\xi}).$$

1) Wien, HARMS, Handb. d. Exp. Phys. 14, 363.

From this it is clear, that the back diffusion cannot explain the deviation from the straight line, because it gives a deviation in the opposite direction.

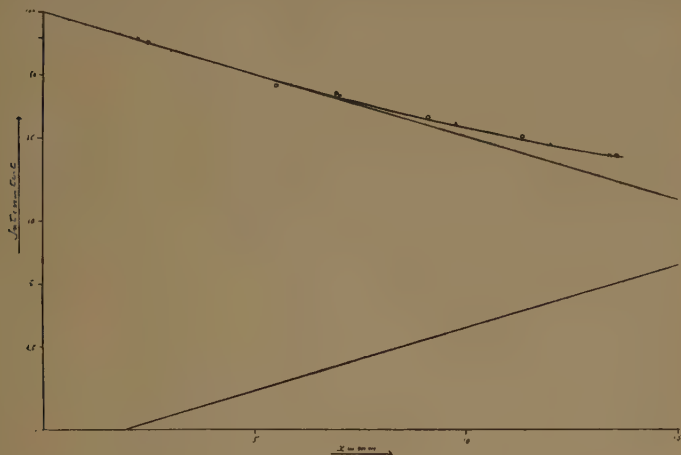


Fig. 1.

Also the assumption that a part of the electrons is scattered backwards and a part forwards cannot explain the effect.

A further possibility of explanation lies in the reflexion of electrons from the surface and within the metal (in our case *Ni*) at the end of the cylinder. For 30 Volts electrons the total reflexion is about 50 % ¹⁾.

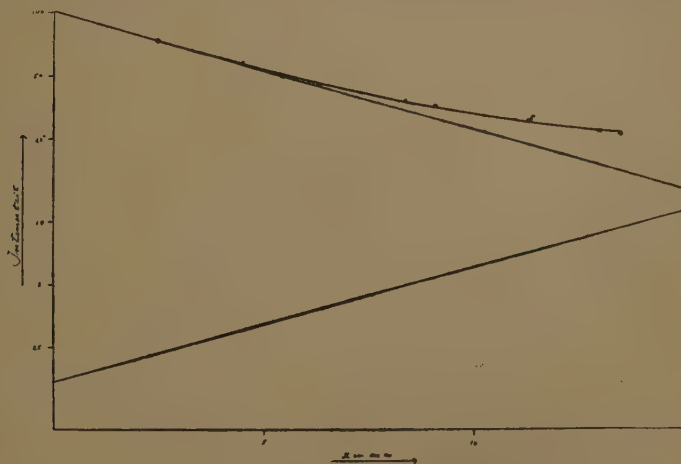


Fig. 2.

¹⁾ Wien, HARMS, Handb. d. Exp. Phys. 14, 342.

We assume thus that the secondary current from the end of the cylinder is half that of the incident current, and that this electron current is reduced in the same way as the primary current.

If we now add the densities of the original and reflected currents we find the points which are shown in fig. 1 by crosses. As is seen, these crosses lie on the experimental curve.

In fig. 2 the same is shown for 76 Volt. Here we must assume a reflexion of 80 %, which is in good agreement with previous experiments. The current density therefore falls off on account of collisions with *He* atoms in exponential form. From the straight line of Fig. 1 we find for the mean free path for 30 Volt electrons in helium at a pressure of 0.1 mm.

$$\lambda_{30} = 0.72 \text{ cM.}$$

and from the straight line in fig. 2

$$\lambda_{76} = 0.78 \text{ cM.}$$

The fact that the mean free path for 76 Volt electrons is greater than that for 30 Volt electrons agrees with the experiments of RAMSAUER.

§ 3. *Measurements with a cylinder of 35 mm. length and a He pressure 0.1 mm.*

In order to test further whether the deviation from the straight line is due to reflexion of the electrons, measurements were made with a longer anode cylinder. The pressure was again 0.1 mm.

One can expect that in this experiment the deviation of experimental curve from the straight line is first to be seen at greater values of the abscissa (it should appear at about the same distance from the end of the cylinder).

The result for 30 Volt electrons is shown in fig. 3 (here the mean value of different lines is again used). We see here that the points lie on a straight line to within 20 mm. from the grid. This shows thus that the deviation from the straight line is indeed not dependent on the action of the gas, but on the dimensions of the cylinder.

At the beginning of the cylinder a noteworthy effect is to be seen. The intensity of the lines first increases and then falls off exponentially. This fact can be explained if one assumes that a diffusion of the field takes place through the grid. At the beginning of the cylinder the electrons have a smaller velocity than that of 30 Volts, and therefore have a smaller probability of causing excitation. As is shown in a former publication, the excitation function of the lines falls off rapidly under 30 Volts. If we assume that the diffusion of the field has a value 4 Volts then this effect is explained. We can test this explanation by an experiment with 36 Volts electrons. Because the excitation curves does not increase for all lines from 32 to 36 Volts this effect, if we take a mean over all the lines, must no longer be observed, and this is actually the case.

In order to determine the electron-density from the intensity we must take account of this alteration of the electron velocity from 32 to 36 Volts for values of x from 0 to 8 mm. (we see from fig. 3 that the diffusion takes

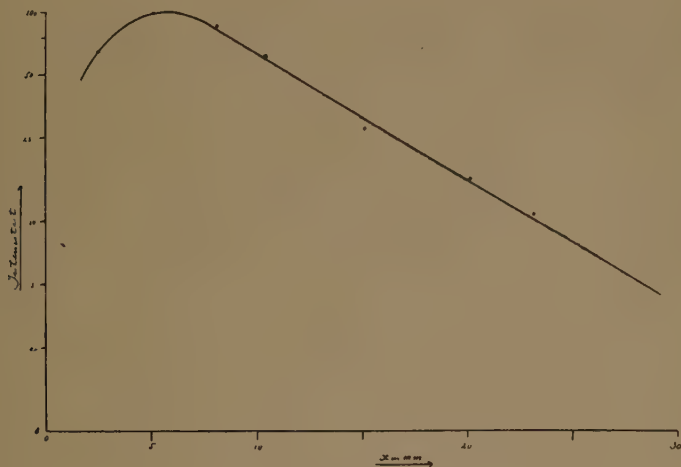


Fig. 3.

place in this region). With the aid of the determined excitation function we can correct the electron-densities.

Further we must make a second correction. Above we have determined the current-density from the intensity, by assuming a linear relation between these variables. The intensity is however, not proportional to the current-density. We must thus make a graph of the intensity and current-density (obtained by currentvariation) and so read off the current density.

Table II gives numbers for 36 Volts electrons corrected in this way. We have therefore found current densities as a function of the distance from the grid, and these must be the same for all lines. That this is not the case must be due to experimental errors. The deviation of the mean values show no relation with the character of the excited level, so that the mean value is approximately the real current-density.

In Fig. 4 this current-density is plotted as function of the distance from the grid (x).

From the straight line it follows that after 20 mm. the current-density is 0.058 of its original value.

The mean free path follows thus from the formula :

$$e^{-\frac{2}{\lambda}} = 0.058$$

$$\lambda = 0.70 \text{ cM.}$$

In order to be able to compare these values with those of RAMSAUER, we calculate from this mean free path the cross-section sum for all atoms in

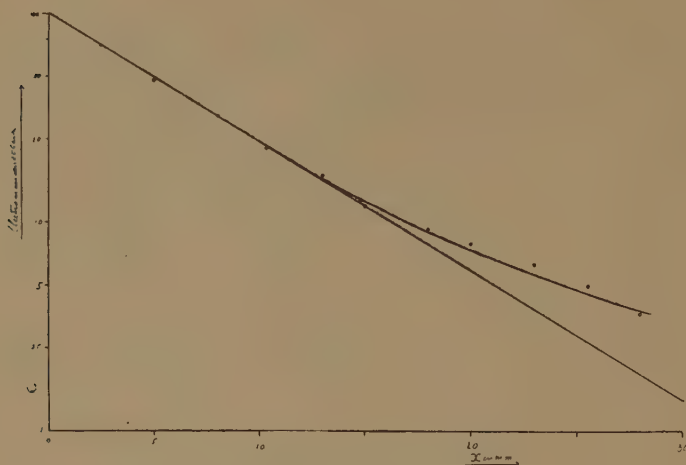


Fig. 4.

TABLE II.

$\alpha^\circ \backslash \lambda$	0	2.5	5	8	10.3	13	15	18	20	23	25.5	28
6678	100	64	45	32	23.5	17.4	13.6	10.8	9.35	7.65	6.2	4.7
5876	100	69	45	30	20	14.2	10.2	7.3	5.3	4.1	2.85	2.14
5016	100	66	43	23.5	13.5	9.0	7.2	5.25	4.05	3.2	2.2	1.44
4922	100	69	52	31	22	16.8	12.8	11.2	9.8	7.8	6.4	5.05
4713	100	81	56	37	25.5	17.4	13.3	11.3	9.1	7.2	5.6	4.0
4472	100	75	46.5	29.5	18.5	12.3	8.3	6.4	5.1	4.0	3.1	2.25
4438	100	71	50	34.5	26.7	20.0	15.1	12.5	10.9	9.6	7.55	5.9
4388	100	76	54	36.5	25.2	21.0	13.9	11.4	10.4	7.9	6.2	5.05
4121	100	69	47.5	36	28.6	21.0	14.0	9.75	7.75	6.35	4.5	2.65
4026	100	72	47	34	24	19.4	12.2	9.9	8.5	6.3	5.2	3.9
3965	100	71	48	30	22	17.2	13.3	9.65	8.80	7.0	5.6	4.0
3889	100	69.5	45	29	20.4	14.4	9.3	6.7	5.75	4.65	3.35	2.53
Mean	100	71.0	48.1	31.9	22.5	16.7	11.9	9.35	7.89	6.31	4.88	3.63

1 cc. of helium at 1 mm. pressure (the value "a" in the work of RAMSAUER).

The effective cross-section of an atom follows from the equation

$$\pi r^2 = \frac{1}{\lambda N}$$

where N is the number of atoms in 1 cc. helium at 0.1 mm. pressure and λ the m.f.p. at this pressure. The number of atoms in 1 cc. at 1 mm. pressure is therefore $10 N$ and the cross-section sum of all the atoms in 1 cc. at 1 mm. pressure is thus :

$$10 N \pi r^2 = \frac{10}{\lambda} = \frac{10}{0.7} = 14.3 \frac{\text{cm}^2}{\text{cm}^3}.$$

It follows from the curves of RAMSAUER that the cross-section for this velocity is about $9 \frac{\text{cm}^2}{\text{cm}^3}$.

This method can in principle also be used to determine cross-sections by electron velocities smaller than the excitation potential of the gas. One can proceed as follows :

In helium at e.g. 0.1 mm. pressure one introduces a small quantity of another gas with a smaller excitation potential. The pressure of this foreign gas must then be so small that the number of electron collisions with the foreign gas is negligible in comparison with the number of collisions with helium atoms. The electron-density is then determined by the *He* atoms and measured with the lines of the foreign gas.

Chemistry. — *2-Decenal-1.* The principal constituent of the essential oil of *Achasma Walang Val.* By P. VAN ROMBURGH.

(Communicated at the meeting of November 30, 1929).

By courtesy of Ir. H. L. WELTER, Director of the Division of Industry, Government Department of Agriculture and Commerce, Buitenzorg, I received on my last visit to Java some samples of the essential oil from the leaves, the petioles and the rhizomes of *Achasma Walang Val.* The odour of these oils resembles very much that of the *walang sangit* (*Leptocorisa varicornis*) a bug living on young riceplants in Java.

According to DE JONG ¹⁾ and to WELTER ²⁾ these oils consist mainly of aldehydes.

From the three samples, I could isolate by repeated careful fractional distillation in vacuo a principal fraction forming a colourless liquid with boiling point of 104° (13 mm.).

At ordinary atmospheric pressure it boils at 229—231°.

Analysis: Found: C 77.78, H 11.80. $C_{10}H_{18}O$ requires C 77.92 H 11.69
 $d_{15}^{19} = 0.846$, $n_D^{19} = 1.4538$.

MR_D found 49.27; calc. for $C_{10}H_{18}O$ 47.87.

This substance shows typical aldehydic reactions. Heated with an ammoniacal solution of silver oxide it deposits a silver mirror.

With semicarbazide in alcoholic solution it yields a semicarbazone which after crystallisation from alcohol formed colourless glistening plates, m.p. 162°.

Analysis: Found: N 19.77, 19.86. $C_{11}H_{20}N_3O$ requires N 20.0.

Though according to the results of the work described in this communication the aldehyde is unsaturated it does not give a colouration with tetranitromethane.

Oxydation of the aldehyde with potassium permanganate.

To 11 g. of the aldehyde suspended in a potassium carbonate solution was slowly added at room temperature 4 % potassium permanganate solution till a permanent pink colour persisted. 50 g. of permanganate were needed.

¹⁾ Jaarboek v. h. Departement van Landbouw, Nijverheid en Handel in Ned.-Indie 1911 p. 46.

²⁾ Ibid. 1921 p. 76.

The solution was then filtered from manganese dioxide and the filtrate acidified with sulfuric acid. An oily liquid separated which was extracted with ether. The extract was dried with anhydrous sodium sulfate. After removal of the ether from the extract the residual liquid was fractionated. The main fraction, having a faint unpleasant odour reminiscent of fatty acids, boiled at 234° and crystallised on cooling in ice. The melting point was 13.5° .

The analysis of the silver salt gave figures in good agreement with the formula of octoid acid $C_8H_{16}O_2$.

Analysis : Found : Ag 42.98 . $C_8H_{15}O_2$ Ag requires 43.03 Ag.

By heating with aniline the acid was converted into its amide m. p. 50° . The compound was identical with the amide from caprylic acid and showed no depression of melting point in admixture.

Hydrogenation ¹⁾ of the unsaturated aldehyde.

5.3 g. of the aldehyde dissolved in neutral ethyl acetate were hydrogenated catalytically in the presence of 0.1 g. of platinum black.

At room temperature only a little hydrogen was absorbed ; on warming and on activating the catalyst with air addition of 2 mol of hydrogen (1500 cc.) was complete in the course of some days.

After evaporation of the ethyl acetate the reduction product was fractionated.

The main fraction consisted of a colourless liquid which almost distilled at $228-231^{\circ}$ and was identified as decanol.

With α -naphthylisocyanate it produced an α -naphthylurethane, which crystallised from petroleum ether (b. p. $40-60^{\circ}$) in white needles m. p. 70° .

The compound was identical in all respects with the naphthylurethane which I prepared from pure decanol.

By heating with 2-3-4-6-tetranitro-1-methylnitraminobenzene the decanol gave yellow crystals of 2-4-6-trinitro-3-oxydecyl-1-nitraminobenzene m. p. 76° .

Both products did not give a m. p. depression when mixed with the corresponding derivatives prepared from pure decanol.

With platinum oxide the hydrogenation gave the following results. In a first experiment 11 g. of the aldehyde (four times distilled in an CO_2 -atmosphere) at 12 mm. in ethyl acetate with 0.1 g. Pt O_2 were shaken with hydrogen. At room temperature 500 cc. of hydrogen were absorbed per 12 hours with constant velocity till 1590 cc. — corresponding with 1 mol H_2 per mol of the aldehyde — had been taken up. Prolonged treatment with hydrogen caused no further absorption. After removal of the solvent I obtained by fractional distillation in vacuo, besides a liquid

¹⁾ I desire to express my thanks to Mr. A. G. VAN VEEN D.Sc. for help in the catalytic reduction of the aldehyde.

portion, as main product a crystalline substance which proved to be decoic acid m. p. 30.5°.

MW (by titration) found: 172.4.

MW $C_{10}H_{20}O_2$ 172.

In a second experiment 8 g. of the aldehyde in ethyl acetate solution with 0.1 g. of platinum oxide absorbed at room temperature 1150 cc. of hydrogen. Then air was admitted into the flask but further shaking and the application of heat were ineffective in causing further hydrogenation.

On distillation at 13 mm., after removal of the solvent, I collected a liquid portion with a boiling point ranging from 100—105°, then the temperature mounted quickly and decoic acid passed over.

36 g. in 4 portions were treated in the same manner.

The liquid distillate was fractionated and a constant boiling fraction obtained (b. p. 99° at 15 mm.) having the characteristics of a saturated aldehyde. It was identified as decanal by its semicarbazone, melting at 99—100° and by oxydation to decoic acid.

Oxydation of the unsaturated aldehyde.

A steady current of oxygen was passed through the aldehyde till the absorption had practically ceased.

The reaction mixture was shaken with a solution of sodium carbonate and extracted with ether to remove neutral products present.

The alkaline portion was acidified with sulfuric acid and the acid removed by ether extraction.

After removal of solvent the acid was subjected to fractional distillation at 15 mm., which gave as main fraction a colourless liquid b. p.¹⁵, 165°. On cooling in the icechest the liquid solidified. M. p. 8°.

MW (by titration) 169.8 MW $C_{10}H_{18}O_2 = 170$.

The silver salt was prepared and analysed

Analysis: Found: 38.89 Ag. $C_{10}H_{17}O_2$ Ag requires 38.96.

The study of the properties of this unsaturated acid is being continued.

Botany. — *The sexuality of Coprinus fimetarius.* (Preliminary note.)

By A. J. P. OORT. (Communicated by Prof. F. A. F. C. WENT.)

(Communicated at the meeting of December 21, 1929.)

Introduction.

The haploid mycelia developed from the spores of a single fruiting-body of quadrisexual Hymenomycetes can be divided into four sex-groups. The discrimination of these four groups depends upon the fact that in certain combinations copulation occurs. As a consequence of this copulation a diploid mycelium is formed, which cytologically is characterized by conjugated nuclei, morphologically by clamp-connections. Following the nomenclature of KNIPE we shall designate these sex-groups by the characters AB , Ab , aB and ab , assuming that there are two pairs of sex-factors: $A—a$ and $B—b$. Copulation and the ensuing diploid mycelium occur exclusively in those combinations in which the components have no common factor. Consequently the diploid mycelium is always heterozygous in both factors ($AaBb$).

My experiments have shown that there are distinctly visible differences in habit between the various haploid combinations. Considering all possible combinations (including the copulating ones) we can with the aid of the above mentioned differences distinguish four combination-groups.

I. There is no difference between the components. This group contains the combinations of mycelia belonging to the same sex-group: $AB \times AB$, $Ab \times Ab$, $aB \times aB$ and $ab \times ab$. In the following these will be called O combinations.

II. The components differ in the A factor: $AB \times aB$ and $Ab \times ab$. We shall designate these combinations by A .

III. The components differ in the B factor: $AB \times Ab$ and $aB \times ab$. These combinations will be called B combinations.

IV. The components differ in both factors: $AB \times ab$ and $Ab \times aB$. In these combinations copulation takes place and a diploid mycelium is formed. They will be called AB combinations.

The distinction of these combination-groups rests upon:

1. Differences in habit.

2. Formation or non-formation of fructifications.

Before going into these differences, it is necessary to say a few words about the circumstances under which the experiments were made.

Method.

I worked with a strain of *Coprinus fimetarius* isolated from germinating seeds of Beta. For studying the habit I used Petri dishes with sterilized horse dung. In order to get a more or less level nutrient medium the dung was loosened and distributed as equally as possible over the dish. With a dish fitting well into the Petri dish (e.g. the cover of a smaller dish) the dung was pressed together and after this a little water was added. The combinations were made by putting together two strips of agar covered with the mycelia in the form of a cross. It is necessary to make certain that the ends touch the nutrient medium, in order to prevent desiccation and death of the mycelium of the strip. The temperature for the cultivation was kept approximately between 23°—25°. In order to get good results (that is, to get evident differences), it is advisable to pay special attention to the following points:

The mycelia which are to be combined, must have been transferred recently (5—10 days ago) and thus be in a state of rapid growth. It is also desirable that the two mycelia have been transferred at about the same time. As any small impurity has an influence on the formation of air-mycelium, it is necessary to work with absolutely pure cultures. For the same reason the humidity of the medium must not be too great. In order to eliminate differences of humidity and constitution of the medium, cultures which are to be compared should be made on the same Petri dish.

In order to study the fruiting-bodies I used tubes with sterilized horse dung. The cultures were kept in daylight, the temperature varying from 15°—25°.

Habit.

We now pass to the discussion of the results, first of all to the differences in habit.

The *O* combinations form a smooth white air-mycelium with downy edge, the habit of which generally agrees completely with that of the haploid mycelium. To some extent this was to be expected. For the haploid mycelium can also be considered as a combination, if it be a combination of two parts of the same mycelium, whereas a *O* combination consists of two different mycelia of the same sex-group. In the following the haploid mycelia will be designated by *OO*, to discriminate them from the real *O* combinations. All the *OO* combinations show the same appearance: there is no morphological sex-differentiation. Thus an *AB* has the same habit as for instance *Ab* or *ab*. The same applies to the real *O* combinations. As far as outward appearance is concerned, *AB* × *AB* cannot be distinguished from *Ab* × *Ab* or *ab* × *ab*. In a few cases a faint line of separation can be seen where the two mycelia are growing together. (Fig. 1: *g* and 3: *h*.)

The components of the *A* combinations ($AB \times aB$ and $Ab \times ab$) seem to check each other, which is evidenced by:

1. retarded growth. (compared with a *O* combination.)
2. formation of fewer air-mycelium; for this reason the culture is more grey in appearance than a *O* combination. Further evidence has been obtained that the two mycelia penetrate each other and thus form a mixed mycelium. The differences between the *A* and *O* combination are sometimes very distinct, sometimes less so. I do not know whether this is caused by individual variations or by differences in constitution or humidity of the medium. (Fig. 1: *a* and *d*, 2: *a*, *f* and *h*, 3: *c*.)

The components of the *B* combinations ($AB \times Ab$ and $aB \times ab$) repel each other; consequently there is a distinct line of separation between the mycelia. With our method of combining, these lines of separation form an easily observable cross. A mixed mycelium is not formed. For the rest the habit is the same as that of the *O* combinations as regards rapidity of growth and quantity of air-mycelium (white colour). (Fig. 1: *c* and *f*, 2: *c*, *d* and *g*, 3: *a*.)

The components of the *AB* combinations ($AB \times ab$ and $Ab \times aB$) form the diploid mycelium, which differs from the *O* combination in greater rapidity of growth and in a contour of apparently radial structure. The greater rapidity of growth cannot or hardly be seen on the pictures because it takes some days before diploid mycelium begins to develop, whereas the haploid mycelium starts at once. (Fig. 1: *b* and *e*, 2: *b* and *e*, 3: *e* and *g*.)

The differences described were obtained with a great number of mycelia.

We can discuss the results from two points of view. Comparing for instance a *B* combination $AB \times Ab$ with the *OO* combination $AB \times AB$ we can say that the difference is caused by the inequality of the *B* factor. If however we compare it with the diploid mycelium $AB \times ab$ we can say that the copulation is prevented by the equality of the *A* factor. The reason of the peculiar habit can thus be sought in the equality of the *A* factors or in the inequality of the *B*'s. It has not been possible to decide which of the two is the principal cause, but the first point of view seems to be more probable. In any case it is now certain that the *A* and the *B* factor do not act in the same way, though we cannot decide to which of the two the observed properties belong.

In order to ascertain that the observed peculiarities were not limited to the strain used, I have made similar experiments with the mycelia of another strain. These were found to lead to the same results.

Other quadrisexual species have not yet been investigated. There are indications that *Coprinus micaceus* behaves at least partly in the same way as *C. fimetarius*. I derived these indications from a table of VANDENDRIES (1927, p. 16, table 7).

Differences in habit (except *C. micaceus*) have not previously been found. However there have been found some other differences between

the *A* and the *B* factor. KNIEP (1928, p. 416) says that with *Schizophyllum* commune the *A* factor is more inclined to mutations than the *B*. BRUNSWIK (1924, see also KNIEP 1928, p. 410) has found something similar with *Coprinus picaceus*. The so-called "Durchbrechungskopulationen" occur in certain combinations (viz. equality of *B* or *b* factor), not in the other ones (viz. equality of *A* or *a* factor).

Fructification.

We shall now consider some other differences between the various combinations, namely concerning the fructification.

Fruiting-bodies occur only very rarely with the strain investigated in the *OO*, *O* and *A* combinations. The fructifications of the *B* combinations occur fairly regularly. However it should be noted that some mycelia are more inclined to form pilei than others. If for the present we limit ourselves to the fructifications of the *B* combinations it may be remarked that the fruiting-bodies of the *B* combinations differ in several respects from the diploid ones. For this reason we have tentatively called them haploid pilei. The differences are so pronounced that one can easily distinguish the two kinds of fruiting-bodies. So far as we have studied them, all haploid pilei formed well germinating spores, though in much smaller quantity than the diploid ones. By an analysis of the spores it should be possible to prove whether we are dealing with real haploid fruiting-bodies, perhaps with a kind of illegitimate diploid fructifications.

When we investigate the spores of a pileus grown from a *B* combination it appears that two kinds of spores have been formed and that these belong to the sex-groups of which the combination was composed. In one case the four spores of one basidium could be isolated. They appeared to belong to one sex-group. Thus it has been shown that we are dealing with haploid fruiting-bodies, though of a peculiar kind, which we must consider as built up of two kinds of hyphae. Both kinds of hyphae form basidia of which, accordingly, we get two kinds. For instance the combination $AB \times Ab$ forms some basidia with the spores *AB*, *AB*, *AB* and *AB* and other basidia with the spores *Ab*, *Ab*, *Ab*, and *Ab*. We might, therefore, regard these pilei as a sort of haploid chimaerae.

The single *A* combination of which a fruiting-body was examined also appeared to form two kinds of spores.

Clamp-connections and mutations.

A closer inspection showed that there were some complications. If we make polysporous cultures of the fruiting-bodies of a *B* combination, these polysporous mycelia (consisting of two kinds of mycelia which together form a *B* combination) make clamp-connections to some degree. Often the clamp-connections are so-called "Pseudo-Schnallen", but many others are quite normal in appearance. The whole phenomenon shows considerable

resemblance to the "Durchbrechungskopulationen" found by BRUNSWIK l.c. in the case of *Coprinus picaceus*. Now a polysporous culture from a fruiting-body of a *B* combination can in itself be considered as a *B* combination, in which the combination took place before the germination. But a disadvantage of these cultures is that we start from a mixture of so many spores (there might perhaps be mutated spores). In order to avoid this I made the spores of a *B* fruiting-body germinate in drops of dung agar in groups of two. The spores were cultivated in moist rooms. In this way I could observe under a microscope that both spores had been germinated. In this case one has to expect 50 % *O* combinations and 50 % *B* combinations. Through re-combination of these bisporous mycelia with the four test-mycelia it is possible to decide whether one has a *O* or a *B* combination. Nearly half of these bisporous mycelia, which later on appeared to be the *B* combinations, showed the following peculiarities:

1. The mycelia did not repel each other so that there were no lines of separation (as in a normal *B* combination).

2. There were spots in the apparently normal haploid mycelium which attracted the attention by a strong formation of air-mycelium, that is by whiter colour.

In some cultures these spots formed sectors, in others they were found near the centre. On studying these whiter spots under the microscope it appeared that in these spots the mycelia formed clamp-connections to some degree, just like the mycelia of the polysporous cultures. The normal mycelium did not show any clamp-connections. Upon transferring parts of the normal mycelium another normal mycelium was formed. By transferring parts of the white sectors a mycelium was formed which at the edges grew normally but at the centre of which a whiter spot with clamp-connections was produced.

In no case did these bisporous or polysporous cultures form diploid fructifications.

Like BRUNSWIK we have thus found "Durchbrechungskopulationen" which in my opinion must be understood in this way: Copulation occurs illegitimately, but no normal diploid mycelium is formed and therefore we do not find any diploid fruiting-bodies. This is not the place for an extensive discussion of the literature, but I want to draw attention to the fact that these experiments have shown most clearly that it is *not* justifiable to speak of mutations when clamp-connections occur in combinations in which, according to the scheme, clamp-connections ought to be absent. As a criterion for diploid mycelia, the occurrence of clamp-connections can only be of a restricted use. Used in connection with the characteristic diploid habit and the diploid fruiting-bodies it remains, of course, a valuable criterion. Presumably many cases which in the literature were described as mutations will appear to be nothing but "Durchbrechungskopulationen". Real mutations probably are much rarer than has been hitherto assumed, whereas illegitimate copulations would seem to occur frequently.

LITERATURE.

BRUNSWIK, H. 1924. Untersuchungen über die Geschlechts- und Kernverhältnisse bei der Hymenomyzetengattung *Coprinus*. Botan. Abhandl. 5.

KNIEP, H. 1928. Die Sexualität der niederen Pflanzen.

VANDENDRIES, R. 1927. Nouvelles recherches expérimentales sur le comportement sexuel du *Coprinus micaceus*. Mém. de l'Acad. Roy. Belg. 2e série, 9: 128.

EXPLANATION OF THE FIGURES.

The photographs were taken 4—6 days after transferring.

Fig. 1: a. A combination $AB \times aB$.

b. AB combination $AB \times ab$.

c. B combination $aB \times ab$. (One of the components somewhat impure).

d. A combination $Ab \times aB$.

e. AB combination $Ab \times aB$.

f. B combination $AB \times Ab$.

g. OO combination $AB \times AB$.

Fig. 2: a. A combination $AB \times aB$.

b. AB combination $AB \times ab$.

c. B combination $AB \times Ab$.

d. B combination $aB \times ab$.

e. AB combination $Ab \times aB$.

f. A combination $Ab \times ab$.

g. B combination $AB \times ab$. (One of the components grows faster; it has been transferred more recently).

h. A combination $AB \times aB$.

Fig. 3: a. B combination $aB \times ab$.

c. A combination $aB \times AB$.

e. AB combination $AB \times ab$. (Partially impure).

g. AB combination $Ab \times aB$.

h. O combination $aB \times aB$. (With a faint line of separation; cf the B comb. fig. 3: a).

Utrecht, November 1929.

Botanical Laboratory.

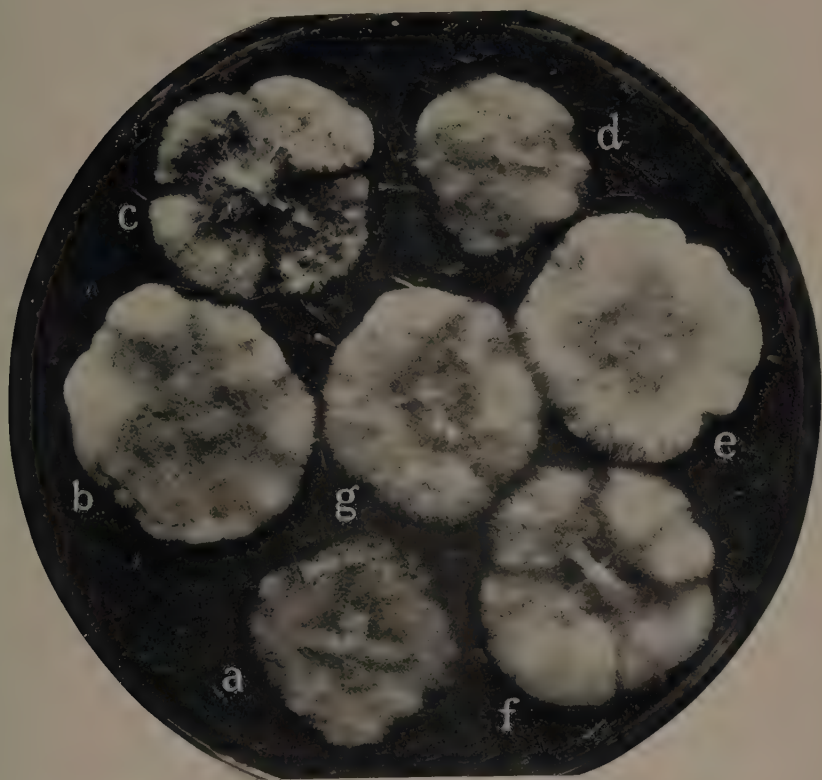


FIG. 1

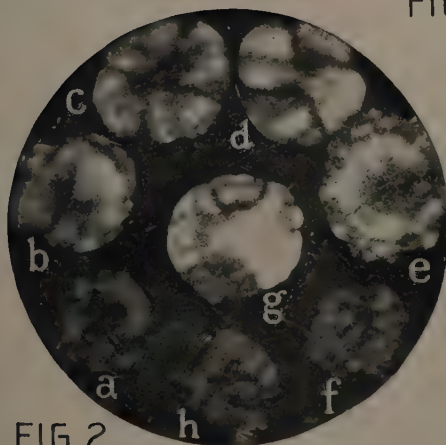


FIG. 2

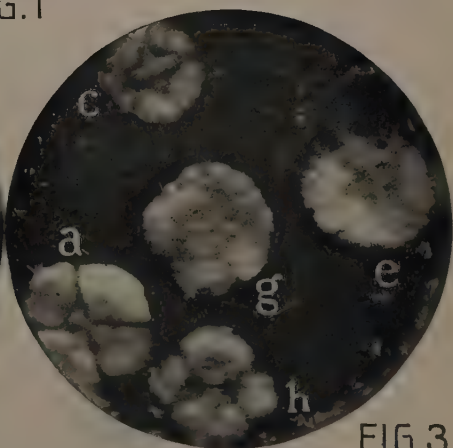


FIG. 3

Botany. — *Ferns of Bawean.* By O. POSTHUMUS. (Communicated by Prof. J. C. SCHOUTE.)

(Communicated at the meeting of December 21, 1929).

The following survey of the Ferns of Bawean is based upon specimens collected by J. D. DORGELO (at Sourabaya) in June 1924, by a native collector of the Buitenzorg Herbarium during a stay of Dr. DAMMERMAN in May 1928 and by the present author in the second half of October 1928¹).

The island is situated in the Java-Sea at 5° 5' southern latitude and 5° 50' east of Batavia, about 160 km. N. of Sourabaya. Its dimensions are in the direction north-south and east-west 13.6 km. and 18 km. respectively.

The island being an old much eroded volcano²) is mountainous, without however a distinct peak dominating the surrounding country. A number of summits, the highest one reaching up to 650 m. are found in a ridge running about 3 km. north of the principal village Sangkapoera and further towards the east running in a northern direction along the eastern border of the crater lake (Telaga) Kastobo. Two of these summits are higher than 600 m., seven of them surpass 550 m. The country east of the Telaga Kastobo is partly less mountainous. Along the coast plains of some size are found only near Sangkapoera on the southern coast and near Tambak on the northern coast; they consist of a rather light alluvial clayey soil. In the other parts of the island the soils have been formed partly by material from the vulcanic rocks, partly from sediments of tertiary age. A peculiar feature in the landscape is the presence of many isolated peaks, partly forming small islets around the island, partly lying isolated in the coastal plains; they are, part of them at least, of vulcanic origin. The island has been raised since the tertiary period, as is shown by the different level at which the horizontal complexes of limestone are found on the slopes of the mountains e.g. the G. Batoe (300 M.), another one south of the village Gunung Malang (100 m.), and near Tandjong Lajar (40 m.).

Some features of the climate are clearly shown by the vegetation. The occurrence of *Gleichenia linearis*, *Lindsaya decomposita*, *Lindsaya repens* and *Pteridium aquilinum* is an indication of a climate without a strong dry season. In Eastern Java these plants do not occur in the dry plains; they are known there only from the moister mountainous regions, the lowermost

¹) The specimens can be found in the Herbarium, Buitenzorg (indicated by H.B.), the Sugar Experiment Station, Pasuruan (POJ) and in my own collection (P).

²) See for a detailed geological description: VERBEEK en FENNEMA, *Geologische beschrijving van Java en Madoera*, 1896, p. 173—192.

localities known to us being situated there at 800, 400, 800, 500 and 800 m. respectively. The same conclusion may be drawn from the occurrence of *Lindsaya orbiculata*, *Schizoloma ensifolium*, *Pteris semipinnata* and *Hemigramma latifolia*, which are known from Western Java but not (yet) from Eastern Java. I got the impression that the northern coast is a little drier than the southern coast, but have not been able, however, to verify this point.

The central portion is wholly covered with forest. This is regularly cut down and burned, for temporary cultivation. After one, seldom two harvests, the soil is exhausted and young forest appears again; consequently, heavy forest is but rarely found, most of the secondary forest being rather young. By its mode of origin it is rather homogenous, the trees in the older portions being all of about the same size, with but scanty undergrowth. In the lower parts *Nephrolepis hirsutula* and *Selaginella plana* occur abundantly; on the ridges *Dryopteris megaphylla* is rather frequent here and there. *Hymenophyllaceae* have not been found.

Perhaps — though I do not think this very probable — some remains of the primary forests may be found in the region between the G. Menangis and the Telaga Kastobo.

POLYPODIACEAE.

Dryopteris Anderson.

1. *Dryopteris amboinensis* (Willdenow) O. Kuntze, Rev. Gen. Plant., II, p. 812, 1891¹⁾. In old secondary forest. N. of Sangkapoera, near kpg. Soengeiteroes laoet, ca. 300 m., Posthumus 1327, 22-10-1928, P.P.O.J.; 1328, 22-10-1928, P.; 1368, 24-10-1928, P. — British India, Malay Archipelago.

2. *Dryopteris immersa* (Blume) O. Kuntze, Rev. Gen. Plant., II, p. 813, 1891. On steep shadowed slopes. Along the road to G. Menangis, N. of the cataract at 300 m., Posthumus 1349, 23-10-1928, P.P.O.J. — Malacca, Malay islands, the Philippines, Hongkong, New Caledonia.

3. *Dryopteris megaphylla* (Mettenius) C. Christensen, Index Filicum, p. 277, 1905. In old secondary forest, often frequent, forming an essential part of the undergrowth. Ridge south of the G. Menangis, ca. 550-575, Posthumus 1347, 23-10-1928, P.P.O.J. — Tropical Africa, Mascarenes, Br. India, Ceylon, Malay islands.

4. *Dryopteris mollis* (Jacquin) Hieronymus, Hedwigia, XLVI, p. 348, 1907. On rather light shadowed places on road-sides. In the numbers 1306, 1319 and 1338 the basal lobe of the upper side of the pinnae is well developed. north of Sangkapoera near desa Kadoe-kadoe, ca. 50 m., Posthumus 1319, 21-10-1928, P.P.O.J.; 1338, 23-10-1928, P.P.O.J.; near desa Kaloemppek, ca. 100 m., Posthumus 1306, 21-10-1928, P. — Tropical countries.

5. *Dryopteris pteroides* (Retzius) O. Kuntze, Rev. Gen. Plant., II, p.

¹⁾ According to C. CHRISTENSEN, The Gardens Bulletin, Straits Settlements, Vol. 4, p. 300, 1929, this species has to be named *D. subpubescens* (Blume) C. Chr.

813, 1891. On road-sides, in brushwood, in rather open or lightly shadowed places. N. of Sangkapoera, near desa Kalboeng, ca. 100 m., Posthumus 1314, 21-10-1928, P.POJ; along the road to Soengeiteeroes, near Balebak, ca. 100 m., Posthumus 1356, 24-10-1928, P.POJ. — Tropical Asia and Australia, Polynesia.

6. *Dryopteris setigera* (Blume) O. Kuntze, Rev. Gen. Plant., II, p. 813. 1891. In open localities, on steep road-sides. N. of Sangkapoera between the cataract and the G. Menangis, ca. 200 m., Posthumus 1343, 23-10-1928, P.POJ. — British India, China, Southern Japan, Malacca, Malay islands, tropical Australia, Polynesia.

7. *Dryopteris unita* (Linnaeus) O. Kuntze, Rev. Gen., Plant., II, p. 811, 1891. Poeloe Menoeri, Karta No. 106, 9-5-1928, H.B. — Macarenes, Seychelles, Southern British India, Ceylon, Malacca, Malay islands, Polynesia.

Aspidium Swartz.

8. *Aspidium Leuzeanum* (Gaud.) Kuntze, Bot. Zeitung, p. 474, 1846. In secondary forest, on the bank of a small river. N. of Sangkapoera, near dessa Kalboeng, ca. 100 m., Posthumus 1313, 21-10-1928, P. — British India, South China, Malay islands, Polynesia.

9. *Aspidium variolosum* Wallich, Hooker, Spec. Filicum, IV, p. 51. 1862. In secondary forest. N. of Sangkapoera, near kpg. Soengeiteeroes laet, ca. 350 m., Posthumus 1373, 24-10-1928, P.POJ. — Northern British India, Birma, Malacca, Java.

Leptochilus Kaulfuss.

10. *Leptochilus cuspidatus* (Presl) C. Christensen, Index Filicum, p. 384, 1905¹⁾. In moist shadowed localities. N. of Sangkapoera, near dessa Soengeiteeroes laet, ca. 300 m., Posthumus 1331, 22-10-1928, P.; id., Posthumus 1369, 24-10-1928, P.POJ. — Seychelles, tropical Asia and Australia, Polynesia.

Nephrolepis Schott.

11. *Nephrolepis hirsitula* (Forster) Presl, Tent. Pterid., p. 79, 1836. Between brushwood, on road-sides and in secondary forest, usually very frequent. N. of Sangkapoera, near dessa Kaloempang, ca. 75 m., Posthumus 1303, 21-10-1928, P.; 1305, 21-10-1928, P.; Poeloe Menoeri, ca. 0—150 m., Karta, 105, 9-5-1928, H.B. — Tropical countries.

Davallia Smith.

12. *Davallia denticulata* (Burmam) Mettenius; Kuhn, Fil. Dech., p. 27, 1867. Poeloe Menoeri, 0—150 m., Karta 104, 9-5-1928, H.B. — Tropical Africa, Madagascar, tropical Asia and Australia, Polynesia.

13. *Davallia solida* (Forster) Swartz, Schrader's Journal, 1800²⁾, p. 87, 1801. Epiphyte, locally frequent. Between Sangkapoera and the Telogo (Kastobo), Dorgelo, June 1924, POJ. — Birma, Malacca, Malay Islands, Polynesia, Queensland.

¹⁾ According to Copeland, Phil. Journ. of Sc., Vol. 37, 1928, p. 366, the name of this fern should be *Campium quoyanum* (Gaud.) Copeland.

Microlepia Presl.

14. *Microlepia speluncae* (Linnaeus) Moore, Index p. XCIII, 1857. In brushwood, in shadowed places. Near Balehakgoeng, N.E. of Sangkapoera, ca. 200 m., Posthumus 1325, 22-10-1928, P.P.OJ.; Posthumus 1362, 24-10-1928, P.P.OJ. — Tropical and subtropical countries.

Schizoloma Gaudichaud.

15. *Schizoloma ensifolium* (Swartz) J. Smith, Journal of Bot., III, p. 414, 1841. In open places, also on slopes. N.E. of Sangkapoera, S.W. of Balibakgoeng, ca. 200 m., Posthumus 1322, 22-10-1928, P.P.OJ. — Tropical Africa, Asia and Australia, Polynesia.

Lindsaya Dryander.

16. *Lindsaya cultrata* (Willdenow) Swartz, Synopsis, p. 119, 1806. On steep, moist, shadowed slopes. N. of Sangkapoera, near kpg. Soengeiteroes laoet, ca. 300 m., Posthumus 1372, 24-10-1928, P. — Madagascar, Réunion, tropical Asia and Australia.

17. *Lindsaya decomposita* Willdenow, Spec. Plant, V, p. 425, 1810. On steep moist shadowed slopes. N. of Sangkapoera, N. of Soengeiteroes, ca. 250 m., Posthumus 1333, 22-10-1928, P.P.OJ. — Tropical Asia and Australia.

18. *Lindsaya orbiculata* (Lamarck) Mettenius; Kuhn, Ann. Mus. Bot. Lugd. Bat., IV, p. 279, 1869. Some specimens possess equilateral pinnae, about $2\frac{1}{2}$ —3 cm. long, the veins of which are sometimes anastomosing; they much resemble *Schizoloma ensifolium*¹⁾. In steep, moist, shadowed localities. N. of Sangkapoera, near kpg. Soengeiteroes laoet, ca. 300 m., Posthumus 1329, 22-10-1928, P.P.OJ.; 1370, 1372a, 24-10-1928, P.P.OJ. — Tropical Asia (Malacca, Sumatra, W. Java and Borneo in the Malay islands) and Australia.

19. *Lindsaya repens* (Bory) Beddome, Ferns of Southern India, p. 72, pl. 209, 1863—'65. On moist shadowed slopes. N. of Sangkapoera, N. of Soengeiteroes, ca. 250 m., Posthumus 1332, 22-10-1928, P.P.OJ. — Mauritius, tropical Asia and Polynesia.

Diplazium Swartz.

20. *Diplazium esculentum* (Retzius) Swartz, Schraders Journal, 1801², p. 312, 1803. In moist, shadowed places along small brooks. N. of Sangkapoera, near cataract, ca. 100 m., Posthumus 1342, 23-10-1928, P.P.OJ. — Tropical Asia, Polynesia.

21. *Diplazium polypodioides* Blume, Enumeratio, p. 194, 1828 (If *D. asperum* Blume is considered to be a distinct species, the material belongs to the latter). In secondary forest, in humid, shadowed places. N.E. of Sangkapoera, between Balibak goeng and Soengeiteroes laoet, ca. 225 m., Posthumus 1326, 22-10-1928, P.P.OJ.; near Balibak goeng, ca. 200 m., Posthumus 1361, 24-10-1928, P. — British India, Malay islands, tropical Australia.

¹⁾ See also *SCHIZOLOMA heterophyllum* (Copeland, Phil. Journ. of Science, I, Suppl. 4, p. 148, 1906).

Blechnum Linnaeus.

22. *Blechnum orientale* Linnaeus. Spec. Plant., ed. II, 1535, 1763. On steep slopes, in brushwood. N.E. of Sangkapoera, S.W. of Balibak goe-noeng, ca. 200 m., Posthumus 1324, 22-10-1928, P.P.O.J. — Tropical Asia, Australia and Polynesia.

Stenochlaena Smith.

23. *Stenochlaena palustris* (Burm.) Beddome, Ferns Br. India. Suppl., p. 26, 1876. Climbing between shrubs. Kampong Disalam, N. of Sangkapoera, ca. 30 m., Posthumus 1379, P.P.O.J. — Tropical Asia, Australia and Polynesia.

Hemigramma Copeland.

24. *Hemigramma latifolia* (Meyen) Copeland, Philipp. Journal of Science, Bot. II, p. 406, 1907. On rocks and steep banks. N. of Sangkapoera, desa Kalboeng, ca. 100 m., Posthumus 1316, 21-10-1928, P.P.O.J.; near Kadoek-kadoek, ca. 50 m., Posthumus 1352, 23-10-1928, P.P.O.J. — Sumatra, Western and Middle Java¹⁾, Borneo, Celebes, the Moluccas and the Philippines.

Ceropteris Link.

25. *Ceropteris calomelanos* (Linnaeus) Underwood, Bulletin Torrey Bot. Club, XXIX, p. 632, 1902. On steep slopes, in lightly shadowed places. N. of Sangkapoera, near desa Kalboeng, ca. 100 m., Posthumus 1312, 21-10-1928, P.P.O.J.; N.E. of Sangkapoera, at the road to Soengeiteroes, ca. 200 m., Posthumus 1358, 24-10-1928, P.P.O.J. — Tropical countries.

Onychium Kaulfuss.

26. *Onychium siliculosum* (Desvaux) C. Christensen, Index Filic., p. 20, 1905. In open of lightly shadowed localities, on steep banks. N. of Sangkapoera, near desa Kalboeng, ca. 100 m., Posthumus 1311, 21-10-1928, P.P.O.J. — Tropical Asia.

Adiantum Linnaeus.

27. *Adiantum lunulatum* Burmann, Flor. Ind., p. 235, 1768. On steep slopes, between boulders, in lightly shadowed places. N. of Sangkapoera, in Kpg. Disalam, ca. 30 m., Posthumus 1380, 25-10-1928, P. — Tropical Africa, Asia and Australia, Polynesia.

28. *Adiantum Mettenii* Kuhn, Fil. Afr., p. 65, 1868. N.W. of Sangkapoera, N. of Soewari, along the road to Teloekdjati, ca. 100 m., Posthumus 1400, 26-10-1928, P.P.O.J. — Tropical Africa, Br. India, the Philippines, Celebes, Timor, Bali and Eastern Java.

Pteris Linnaeus.

29. *Pteris biaurita* Linnaeus, Spec. Plant., II, p. 1076, 1753. In brushwood. N. of Sangkapoera, near desa Kalboeng, ca. 100 m., Posthumus 1309, 21-10-1928, P.; between Sangkapoera and Telogo Kastobo, Dorgelo 63, 17-6-1924, P.O.J. — Tropical countries.

30. *Pteris ensiformis* Burmann, Flor. Ind. p. 230, 1768. On steep,

¹⁾ It has been mentioned with doubt for Banjuwangi, one of the comparatively few parts of East Java with a moist climate (Kurz, Journ. As. Soc., Vol. 39, 1870, p. 90).

shadowed road-sides. N. of Sangkapoera, near desa Kaloempang, ca. 75 m., Posthumus 1304, 21-10-1928, P.P.OJ. — Tropical Asia and Australia; Polynesia.

31. *Pteris heteromorpha* Fée, Genera Filicum, p. 127, 1850—1852. On slightly or well shadowed, steep road-sides. N. of Sangkapoera, near desa Kalempak, ca. 100 m., Posthumus 1307, 21-10-1928, P.P.OJ.; near desa Kalboeng, ca. 100 m., Posthumus 1315, 21-10-1928, P.; between Sangkapoera and Telogo Kastobo, Dorgelo, 59, 17-6-1924, P.OJ. — Birma, East-Java, Madoera, Celebes and Luzon.

32. *Pteris quadriaurita* Retzius, Observationes, VI, p. 38, 1791. On road-sides, in shadow. N. of Sangkapoera, W. of desa Kalboeng, ca. 100 m., Posthumus 1318, 21-10-1928, P. — Tropical countries.

33. *Pteris semipinnata* Linnaeus, Spec. Plant., p. 1076, 1753. On steep, slightly shadowed road-sides. N. of Sangkapoera, near kpg. Soengeiteroes laoet, ca. 300 m., Posthumus 1330, 22-10-1928, P.P.OJ.; Posthumus 1371, 24-10-1928, P.P.OJ. — British India, China, Japan, Birma, Malacca, Western Java, Borneo, Celebes, the Philippines.

34. *Pteris vittata* Linnaeus, Spec. Plant., II, p. 1074, 1753. On road-sides. S.E. of Tambak, near desa Lawangan, ca. 40 m., Posthumus 1336, P. — Tropical countries.

Pteridium Gleditsch.

35. *Pteridium aquilinum* (Linnaeus) Kuhn, v. Decken, Reisen, 3⁸. Bot., p. 11, 1879. In open places, or on the margin of brushwood. N. of Sangkapoera, S. of Leban, ca. 300 m., Posthumus 1335, 22-10-1928, P.P.OJ.; N. along the road to Soengeiteroes, ca. 300 m., Posthumus 1360, 24-10-1928, P.P.OJ. — Tropical and temperate countries.

Anthrophyum Kaulfuss.

36. *Anthrophyum callifolium* Blume, Enumeratio, p. 111, 1828. In old secondary forest, epiphytic and on big boulders, in shadowed localities. Near the cataract along the road to G. Menangis, ca. 100 m., Posthumus 1339, 23-10-1928, P.P.OJ.; near kpg. Soengeiteroes laoet, ca. 300 m.; Posthumus 1374, 24-10-1928, P.P.OJ.; between Sangkapoera and Telogo Kastobo, Dorgelo 32, 17-6-1924, P.OJ. — Malay Islands, Polynesia.

Loxogramma Presl.

37. *Loxogramma Blumeianum* Presl. Tent. Pterid., p. 215, 1836. In old secondary forest, epiphytic. Along the road to G. Menangis, N. of the cataract, ca. 400 m., Posthumus 1348, 23-10-1928, P.P.OJ. — Malay Islands, S. China, Japan.

Polypodium Linnaeus.

38. *Polypodium Phymatodes* Linnaeus, Mantissa, p. 306, 1771¹⁾. In stony places, frequent on the walls separating the premises. N.E. of Sangkapoera, kpg. Balibak, ca. 200 m., Posthumus 1363, 24-10-1928, P.P.OJ.;

¹⁾ According to Copeland, Ferns of Fiji, Bernice Bishop Museum Bulletin 59, p. 93, 1929, the name *Polypodium scolopendrina* Burmann should be preferred.

between Sangkapoera and Telogo Kastobo, Dorgelo 56, 17-6-1924, POJ.
— Tropical Africa, Asia and Australia, Polynesia.

Cyclophorus Desvaux.

39. *Cyclophorus adnascens* (Swartz) Desvaux, Berl. Mag., V. p. 300, 1811. On trees, in old secondary forest. N. of Sangkapoera, near kpg. Soengeiteroes laoet, ca. 300 m., Posthumus 1375, 24-10-1928, P.P.OJ. N. of Soengeiteroes, ca. 250 m., Posthumus 1334, 22-10-1928, P. — Tropical Asia, Polynesia.

Drynaria J. Smith.

40. *Drynaria quercifolia* (Linnaeus) J. Smith, Journal of Bot., III, p. 398, 1841. In old secondary forest, on trees. Along the road to G. Menangis. N. of the cataract, ca. 350 m., Posthumus 1345, 23-10-1928, P.P.OJ. — Tropical Asia, S. China, Tropical Australia, Fiji.

41. *Drynaria rigidula* (Swartz) Beddome, Ferns of Br. India, pl. 314, 1869. On trees, in old secondary forests. Along the road to G. Menangis. N. of the cataract, ca. 350 m., Posthumus 1346, 23-10-1928, P.P.OJ. — Tropical Asia, Australia and Polynesia.

Acrostichum Linnaeus.

42. *Acrostichum aureum* Linnaeus, Species Plant., II, p. 1069, 1753. Along the coast, in the Nipa-formation. Near Sangkapoera, along the road, Posthumus 1377, 24-10-1928, P.P.OJ. — Tropical countries.

PARKERIACEAE.

Ceratopteris Brongniart.

43. *Ceratopteris thalictroides* (Linnaeus) Brongniart, Bulletin Soc., Philom., p. 186, 1821. In inundated rice-fields. N. of Sangkapoera, ca. 10 m., Posthumus 1354, P.P.OJ. Karta 166, 13-5-1928, H.B. — Tropical and subtropical countries.

GLEICHENIACEAE.

Gleichenia Smith.

44. *Gleichenia linearis* (Burm.) Clarke, Trans. Linn. Soc. II, Bot. I, p. 428, 1880. Among shrubs. N.E. of Sangkapoera, S.W. of Balibakgoe-noeng, ca. 200 m., Posthumus 1323, 22-10-1928, P.P.OJ. — Tropical countries.

SCHIZAEACEAE.

Lygodium Swartz.

45. *Lygodium circinnatum* (Burm.) Swartz, Synopsis, p. 153, 1806. Climbing, between shrubs. N. of Sangkapoera, near dessa Kalboeng, ca. 100 m., Posthumus 1310, 21-10-1928, P. — Tropical Asia and Australia.

46. *Lygodium flexuosum* (Linnaeus) Swartz, Schraders Journal 1800², p. 106, 1801. Climbing, between shrubs. S. of Tambak, near dessa Lawangan, ca. 40 m., Posthumus 1337, 22-10-1928, P.P.OJ. — Tropical Africa, Asia and Australia.

MARSILIACEAE.

Marsilia Linnaeus.

47. *Marsilia crenata* Presl, Rel. Haenk., I, p. 84, pl. 12, fig. 13, 1825. In inundated rice-fields. N. of Sangkapoera, ca. 10 m., Posthumus 1321, 21-10-1928, P.POJ. — Br. India, Malay Islands.

MARATTIACEAE.

Angiopteris Hoffmann.

48. *Angiopteris evecta* Hoffmann, Comm. Soc. Reg. Gott. XII, p. 29, pl. 5, 1796. In old secondary forest, along river, in shadowed humid places. Along the road to G. Menangis, N. of the cataract, ca. 300 m. (observed 22-10-1928, no specimens collected). — Tropical Asia, Africa and Polynesia.

SELAGINELLACEAE.

Selaginella Spring.

49. *Selaginella plana* Hieronymus in Engler-Prantl, Nat. Pflanzenfam., I, abt. IV, p. 704, 1900. In secondary forest and in brushwood, rather common in slightly shadowed places. N. of Sangkapoera, near desa Kal-boeng, ca. 120 m., Posthumus 1308, 21-10-1928, P.POJ; between Sangkapoera and Telego Kastobo, Dorgelo 62, 17-6-1924, POJ. — British India, Malay Islands.

It is shown by this list that all the species of the Fern-vegetation of Bawean are found on Java too; species occurring in Borneo, but not known from Java, are wholly wanting. But it should not be forgotten, that no primary vegetation could be investigated, it being most probably wholly destroyed, and that ferns occurring in Sumatra, Malacca and Borneo, but not in Java, are nearly all restricted to the primary vegetation¹⁾.

Two species only are not known from Western Java; they show a remarkable area: *Adiantum Mettenii* Kuhn, which is known from Eastern Java²⁾, Bali³⁾, Timor⁴⁾, Celebes⁵⁾, the Philippines, British India and Africa; and *Pteris heteromorpha* Fee known from Eastern Java⁶⁾, Madoera⁷⁾, Celebes, Luzon and Birma. These areas thus remind of the

¹⁾ POSTHUMUS, Notes on Pteridophyta from Djambi, Sumatra. These Proceedings, 31, 1928, p. 111.

²⁾ G. watangan near Poeger, KOORDERS 21194 β (HB).

³⁾ G. Kelatakan, ca. 285 m., Maier-Sarip 163 (HB).

⁴⁾ Saluki, kingdom of Bibiçuçu, ca. 1000 m., Forbes 3753 (HB, cf. Gepp in Journal of Botany, Supplement 1925).

⁵⁾ Falls near Maros, Sarasin 1197 (*Adiantum lunulatum*, var. *Mettenii* Christ, Ann. Jard. Bot., Buitenzorg, 15, 1898, p. 104).

⁶⁾ VAN ALDERWERELT VAN ROSENBURGH, Malayan Ferns, 1906. p. 362.

⁷⁾ Near Bangkalan, BACKER 19067 (HB); near Raparx, 150 m., BACKER 20238 (HB.)

distribution of some other species of ferns, e.g. *Cheilanthes farinosa* Kaulf., *Dryopteris prolifera* C. Chr. and *Athyrium Otaria* Posth. (*Dryopteris Otaria* Kze, *Dryopteris paucijuga* v. A. v. R.), which via the Philippines extend to the eastern part of the Malay Archipelago, including Eastern Java.

The other ferns of this list are all known from Western Java ; some of them have not been found in Middle or Eastern Java. They are the following :

Dryopteris megaphylla (known from West- and Middle Java), *Lindsaya orbiculata* and *Pteris semipinnata* (known from West-Java). The occurrence in Eastern Java of *Schizoloma ensifolium* and *Hemigramma latifolia*, known from West- and Middle Java, is rather doubtful ; the statements in the literature have not been confirmed afterwards. But all these species are widely distributed in the Malay Archipelago, also towards the East, towards New Guinea, the Moluccas and the Philippines some of them even far going beyond.

Physiology. — *The electrogram of the submaxillar salivary gland.* By
A. VAN HARREVELD. (Communicated by Prof. G. VAN RIJNBEEK.)

(Communicated at the meeting of December 21, 1929).

Since the investigations of BAYLISS and BRADFORD dating from about 1885 it is known that stimulation of the chorda tympani and of the sympathetic in the neck is accompanied by changes of potential in the submaxillar gland. Other investigators studied those action currents also, but they did not obtain corresponding results. The differences in the electric phenomena described are partly due to the use of different methods. It is important that some workers made use of instruments with a large inertia for the registration of the potential differences; BAYLISS and BRADFORD using a THOMSON galvanometer, CANNON and MC KEEN CATTEL and also GESELL using a galvanometer after the type of DEPREZ—D'ARSONVAL, while others, BECK and ZBYSEWSKI, RABL and also GAYDA used the string-galvanometer.

In the experiments described below the action current of the gland was registered with the latter type of galvanometer. In each photogram the secretion was registered with a continuous method as well as the change of volume of the gland. BUNCH stated, that the change in volume of the submaxillar saliva gland after stimulation of the chorda is determined by two factors of opposite influence; the secretion causes the volume of the gland to decrease, and the vascular dilatation, also caused by the stimulation of the chorda tympani, results in an increase of volume. If the venous flow is slightly impaired the change in volume caused by the vascular dilatation should be decisive. In the experiments described below the venous flow certainly was not wholly free, since a hollow rubber cuff was placed round the gland with slight over-pressure. The changes in volume as registered, therefore will be chiefly caused by the changes in vascular lumen.

The action current of the submaxillar saliva gland in dog showed a multitude of forms. Not only that stimulation of chorda and sympathetic nerve yielded electrograms of different aspect, but stimulation of one of the nerves showed large individual variations.

Faradic stimulation of the chorda during about 10 seconds resulted after a latent period of an average of 0.3 sec. in electrical changes in the gland, which caused, at least shortly after starting of the stimulus, the surface to become negative in regard to the hilus. The electrogram usually showed several peaks in the first seconds, sometimes preceded by a small peak in opposite direction, the surface being temporarily positive in regard to the hilus. Often the galvanometer remained in rest during the later period of

stimulation, sometimes the surface of the gland became more and more negative during further stimulation, and in other cases the variation in potential changed its direction, the surface of the gland turning positive. In the cases in which the string showed clear deflections in the later period of stimulation, the action current was followed by an after-effect.

The electrical phenomenon resulting from a 10 second faradic stimulation of the sympathetic nerve also showed individual variations. Usually after a latent period averaging 0.8 sec. the surface of the gland turned positive in regard to the hilus. Those changes in potential had a much slower course than those resulting from chorda stimulation. Usually after about 2 seconds the positivity of the surface reached a maximum, and decreased afterwards, sometimes even excessively, causing the string to cross the zero-line, and the surface becoming negative in regard to the hilus. Sometimes the positivity of the surface of the gland increased during the entire time of stimulation. In a few cases the surface immediately became negative. The latent period however, then was longer, and amounted to about 2 sec. No after-effect was ever seen when stimulation of the sympathetic was stopped.

Since the electrical resistance of a tissue is to a certain extent looked on as a measure of the permeability of the walls of the cells, this value becomes of special importance for a gland that absorbs substances during the process of secretion from the blood and also secretes the same, changed or unaltered. It must be born in mind that the ionic permeability only determines the resistance of an organ. BRONK and GESELL registered the changes in resistance with an inert galvanometer. As indicator current an alternating current of 1000 Herz was used. Stimulation of the chorda resulted in an increase in resistance after a latent period of 3 sec. Stimulation of the sympathetic also increased the resistance of the gland somewhat, PESERICO registering the changes in resistance with a string galvanometer, found an initial decrease in resistance stimulating the chorda, when using direct as well as alternating currents of 100.000 up till 1.000.000 Herz. Those changes occurred after a latent period of 0.25 sec. After this initial decrease, the resistance increased again, causing the string to cross the zero-line, and the resistance of the gland becoming larger than in rest. The same course of the changes in resistance during stimulation was seen by me. Stimulation of the sympathetic resulted after a latent period of several seconds in an immediate increase of resistance of the gland.

When the tracings from the action current and from the changes in resistance caused by stimulation of the chorda tympani are compared, a certain analogy in those electrograms after the third or fourth second of stimulation becomes evident. This analogy gave rise to the question: is it possible that a change in resistance in the gland manifests itself in the action current?

The system of gland and string may be compared with a source of current shunted through two circuits, one consisting in electrodes and galvanometer,

the other in the tissue of the gland itself. In the gland a rest-current was seen of various intensity and index. If this change in potential be imagined to take place in a certain level of the gland, this level can be compared with a sieve. The holes in the sieve are represented by the saliva ducts, the substance by the total amount of cells. The substance of the sieve being the seat of the potential difference, this is short-circuited by a conductor consisting in the saliva ducts and partly in the glandular tissue. The resistances of electrodes and string do not change when the chorda is stimulated ; but the second shunt, consisting a.o. in the glandular cells does change. This change in resistance is even considerable ; the total resistance against direct currents being after PESERICO 600 Ohm, I noted in several animals during stimulation of the chorda an increase of 500 Ohm. Since the resistance of the shunt, which short-circuits the rest-potential, becomes about twice its value by stimulation of the chorda tympani, the E.M.F. in the place of the electrodes will increase, therefore changes in resistance will show as a change of the rest-current. Even after compensating the rest-potential, which certainly is less than the true difference in potential by which it is caused, the change in resistance will cause a deviation of the rest-current.

In the cases which showed a clear and constant rest-current this appeared to be analogous with the action current during the last period of stimulation. When no clear rest-potential was apparent, the string remained in rest during this period. Furthermore, if this opinion on this part of the electrogram be true, it must be possible to reproduce the various forms of action currents in the later period of stimulation by sending a constant current through the gland, that will be changed by the variations in resistance caused by chorda stimulation. This turned out to be the case. The variations in resistance being much less pronounced by stimulation of the sympathetic, no influence of change in resistance could be seen in this electrogram.

One of the most marked changes shown by the electrogram during chorda stimulation is caused by the injection of a certain dose of atropin. The discoverers of the gland action current, BAYLISS and BRADFORD, noticed already the change of direction of the current under the influence of this alcaloid. The electrograms of Fig. 1 and 2 represent the action

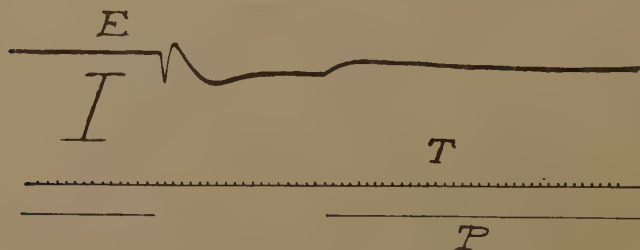


Fig. 1.

currents before and after intravenous injection of $\frac{3}{4}$ mgrs. of atropin sulfate. As is shown by the absence of deflection during the later period

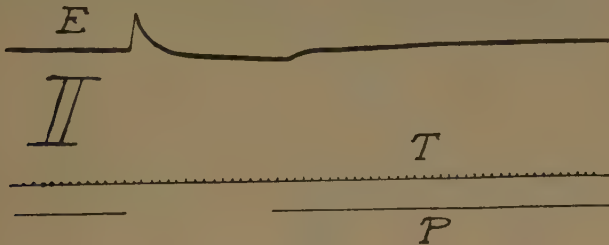


Fig. 2.

of stimulation, the change in resistance of the glandular tissue during chorda stimulation was not apparent in this electrogram. A comparison of both electrograms shows that the atropin has caused a change of direction of the action current and an increase of latent period from $\frac{1}{3}$ to $\frac{1}{2}$ second. This change in the electrogram shows the probability of the action current being a compound of two electric phenomena of opposite direction. Usually the change in potential causing the surface of the gland to become negative should be the larger. The atropin causes the disappearance of the latter part of the action current thus enabling the current of opposite direction to become apparent.

We will call the part of the action current that causes the surface of the gland to become negative in regard to the hilus during chorda stimulation, the "negative" phase, and the part that causes the surface to become positive in regard to the hilus the "positive" phase.

The positive phase can directly be registered from the gland, after the disappearance of the negative phase by the use of a certain dose of atropin, but the aspect of the negative phase cannot be determined in this simple way. But if the conception of the electrogram during chorda stimulation as the compound of two electrical phenomena of opposite direction be true, it must be possible to get an impression of the negative phase by combining the electrogram before the injection of atropin and the negative of the electrogram after the injection. This construction is carried out in Fig. 3. The result is a line of simple form. The action current during stimulation of the chorda therefore appears to be composed of two electrical phenomena of simple

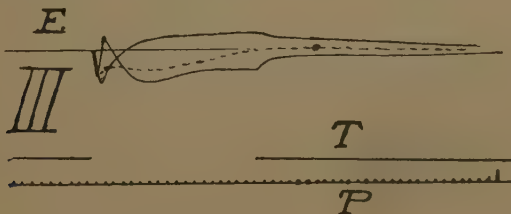


Fig. 3.

course and this can be regarded as an argument in favour of the hypothesis that the electrical disturbances are due to two single processes in the gland.

In most cases the latent period of the negative phase was shorter than the same of the positive phase, the initial deflection therefore showing negativity of the gland surface, but some cases occurred in which the latent period of the positive phase was the shorter. Repeated stimulation of the chorda in one and the same dog changed the latent period of negative as well as of positive phase. In one dog it was noticed that the latent period of the positive phase was originally the shorter, but increased during the experiment, the latent period of the negative phase on the other hand became shorter; this resulted finally in a change of direction of the initial deflection of the electrogram.

The aspect of the change in resistance during chorda stimulation changed too, when a certain dose of atropine was injected. Comparing two resistance curves, one before, the other after the injection of $\frac{3}{4}$ mgrs. of atropin, it appears that the latent period of the change in resistance has changed under the influence of the alkaloid from $\frac{1}{3}$ to $\frac{1}{2}$ sec. In the curve of change in resistance obtained before the injection of atropin, the initial decrease of resistance is followed by a marked increase, but after injection this increase remains absent (Fig. 4 and 5). If the

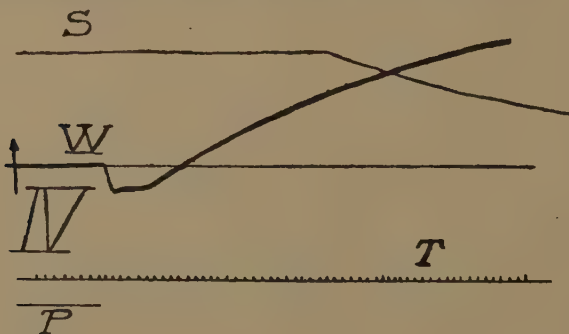


Fig. 4.

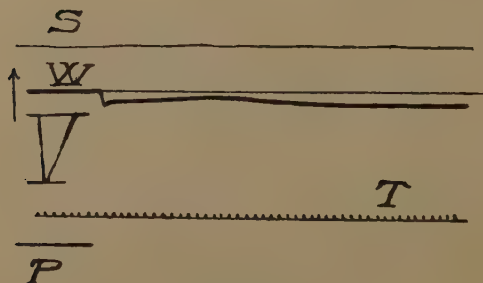


Fig. 5.

attempt is made to trace the parallel existing between action current and change in resistance still further, and to construct a compound curve of the change in resistance before the injection of atropin and its negative after the injection, in order to get an impression of the part destroyed by the alcaloid, a curve is obtained showing a course as irregular as shown by the resistance curve before atropin was administered (Fig. 6).

A priori it seems probable to suppose that, when an electrical phenomenon

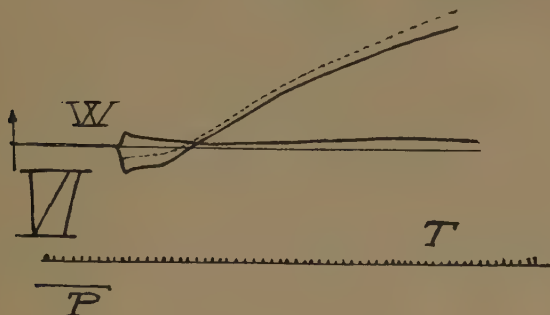


Fig. 6.

is caused by one single process, the curve representing this phenomenon will have a simple course. If this supposition be true, we must assume that the compound curve mentioned above representing the part of the change in resistance that can be destroyed by a certain dose of atropin, cannot be ascribed to a single process in the gland, or that the part of the change in resistance, remaining after atropin and which was supposed not to be altered by the alcaloid, has been altered by the influence of the atropin injected. It will be shown presently that the positive phase and the change in resistance remaining after the injection of atropin do agree in so many aspects, that it is probable to suppose both phenomena to be caused by the same process. Figures 1, 2, 4 and 5 show the effect of an equal dose of atropin on action current and change in resistance during chorda stimulation of the same gland. Since the compound curve of the action current before the injection of $\frac{3}{4}$ mgrs. atropin and the negative of the same after the injection appears as a line of simple form, the positive phase cannot have been altered greatly by the injection. Since the positive phase and the part of the change in resistance remaining after injection of atropin are probably results of the same process, we must assume that the latter part of the change in resistance also is uninfluenced by the injection. We therefore reach the conclusion that the initial decrease in resistance of the non-poisoned gland is the sum of two decreases in resistance. The increase in latent period of resistance variation after injection of atropin supports this conclusion. In the curve of the action current shown in Fig. 1 we see $\frac{1}{2}$ sec. after the beginning of stimulation the influence of the positive phase becoming apparent, by a decrease of the

negativity of the gland surface. This shows that also in a gland unaltered by atropin the positive phase has a latent period of $\frac{1}{2}$ sec. Now, since, as said above, the positive phase and the decrease in resistance remaining after injection of atropin both are supposed to be due to the same process in the gland, the decrease in resistance in the non-poisoned gland appearing after a latent period of $\frac{1}{3}$ sec. cannot be caused alone by the variation of resistance due to the same process as the positive phase. The initial decrease of resistance during chorda stimulation therefore must be caused by two decreases of resistance with a different latent period. The resistance variation during chorda stimulation which disappeared by injection of a certain dose of atropin consisted, as is shown in the compound curve Fig. 6, of an initial decrease of resistance followed by a larger or smaller increase. Since the increase was often very slight in the beginning of the experiment or could not be noted at all, and on the other hand attained often very high values after repeated stimulation, it is probable that those two kinds of variations are to a certain extent independent from each other. We therefore believe that the typical course of the resistance variation that disappears under the influence of a small dose of atropine is caused by a compound of two decreases in resistance reaching its maximum in a few seconds, and of an increase which reaches its apex much later, sometimes even after 40 sec. This latter variation was in most cases much larger than the former. This explains why during the first seconds of chorda stimulation the decrease dominates the increase which grows but slowly, during the latter seconds however the second variation being the larger.

The analysis of action current and change in resistance during chorda stimulation succeeded by reason of the different sensibility of its components for atropin, but the lack of such a substance made it impossible to do the same with the electrogram resulting from stimulation of the sympathetic. The diphasic form of the action current, that was commonest after stimulation of the sympathetic raised the question whether this phenomenon might be caused by several electrical changes in the gland, as is the case with the action current after chorda stimulation. If the monophasic action currents that appear after stimulation of the sympathetic but which are much rarer, can be looked on as consisting of one of the components of the diphasic current, it is possible to obtain the complicated electrogram by summing both the simple curves. The course of the initial deflection is determined by the electrogram with the shortest initial period, this being the change which turns the surface positive in regard to the hilus. In the diphasic electrogram during sympathetic stimulation usually two seconds after starting the stimulus the initial deflection decreases. Two seconds is also exactly the latent period of the monophasic electrogram which turns the surface negative in regard to the hilus. After this opinion the diphasic electrical phenomenon which is usually the result of stimulation of the sympathetic consists of two potential variations of different latent period and different direction. By some reason unknown to us, sometimes one of those phenomena may fail to appear,

which results in the occurrence of the other two types of action current during sympathetic stimulation.

We failed to demonstrate relations between action current and vascular changes as a result from stimulation of the gland-nerves, as neither did BAYLISS and BRADFORD, CANNON and MC KEEN CATTEL. Neither a direct relation between secretion and action current could be demonstrated. We could confirm PESERICO's observation, that there exists no relation between variation in resistance and changes in vascular lumen during indirect stimulation of the gland. No relation could be shown to exist between secretion and change in resistance.

We spoke already of relation between electrogram and change in resistance of the gland. We ascertained that the electrogram registered without further precaution, must be looked on as the compound of the curves of action current and change in resistance in the gland.

It is also possible that for instance one of the resistance variations is related with one of the components of the action current, both electrical phenomena being due to one and the same process in the gland. If this possibility is investigated, it becomes apparent that in the dog which yielded the photograms shown in Figs. 1, 2, 4 and 5, the latent period of the negative phase amounted to about $\frac{1}{3}$ sec., and of the positive phase to $\frac{1}{2}$ sec., both decreases in resistance occurring also after $\frac{1}{3}$ resp. $\frac{1}{2}$ sec. Furthermore, after injection of atropin in a dose of $\frac{3}{4}$ mgrs. the negative phase disappeared as well as the decrease of resistance with the latent period of $\frac{1}{3}$ sec., the positive phase and the decrease in resistance with $\frac{1}{2}$ sec. latent period remaining. After injection of another dose of atropine the electrical phenomena last mentioned disappeared together. This makes it probable that action current and both the variations in resistance during chorda stimulation are caused by not more than two processes in the gland.

PESERICO found that the changes in resistance measured with alternating currents from 100.000 up till 1.000.000 Herz showed the same course, but were less intense than measured with direct current as an indicator. He therefore drew the conclusion that variations in the concentration of electrolytes were not the cause of those changes in resistance, but changes in the mobility of the ions. In each cell membranes are present which are but little permeable for ions. The passing of direct current should cause an accumulation of ions at both sides of such a membrane, representing a potential difference. The thousands of membranes between the electrodes together might be the cause of the potential difference that results in the discrepancy between direct and alternating current resistance as it is measured. From the variations in resistance of the submaxillar gland during stimulation of its nerves we must draw the conclusion, that a change in permeability of its cellmembranes has occurred.

In the arrangement of TARCHANOFF, for studying the psychogalvanic reflex, without an indicator current, an action current is demonstrated. In the arrangement of VERAGUTH in which an indicator current goes

through the skin, a decrease in resistance occurs under the same circumstances, that give rise to an action current in TARCHANOFF's experiment. GILDEMEISTER demonstrated that this decrease in skin resistance must be regarded as an increase of permeability of the cells. PESERICO demonstrated the same to be true in the case of variations in resistance in the submaxillar gland. As in the skin a close relation must be assumed to exist between decreasing polarisation of certain parts of the cell and action current. It may even be supposed that the action current is the direct result of this change of polarisation of the gland cells.

EXPLANATION OF THE FIGURES.

Figs. 1, 2, 4 and 5 show the influence of a small dose of atropin on the action current and resistance variation in the same submaxillar saliva gland during chorda stimulation. Fig. 1 represents the electrogram before, Fig. 2 after injection of the alcaloid. Figs. 4 and 5 represent the resistance variation before and after atropin injection. Figs. 3 and 6 show the compound curves of electrical phenomenon resp. resistance variation before injection, and the negative of those values after injection.

The letters in the figures have the following meaning: The line indicated with "E" represents the action current (A deflection of the string under the zero-line represents an electric phenomenon which turns the surface of the gland negative in regard to the hilus) "T" is the time in seconds, "P" stimulation of the nerve (the interruption represents duration of stimulus) and finally "W" represents variations in resistance (if the galvanometer deflects in the direction of the arrow drawn through this line, the gland resistance increases).

Physics. — *The use of the effect of pressure on the electrical resistance of manganin as a method of measuring pressure.* By A. MICHELS and M. H. LENSSEN. (25th Communication of the "VAN DER WAALS-Fund".) (Communicated by Prof. J. D. VAN DER WAALS Jr.)

(Communicated at the meeting of December 21, 1929).

Introduction.

As a result of work on the influence of pressure on the electrical resistance of gold and platinum ¹⁾ it was decided to investigate the accuracy that could be obtained in measuring pressure by means of this effect. Following BRIDGMAN ²⁾, manganin was chosen because of its small temperature coefficient.

It was first necessary to investigate the treatment required by this material in order to ensure its constancy.

Apparatus.

As with the measurements with gold ¹⁾, the actual resistance of the wire under pressure was not measured but a differential method was used. In this, the ratio of the resistances of the wire under pressure and of a similar wire at the same temperature but not under pressure was determined. No advantage could be gained by measuring the actual resistance of the wire owing to the uncertainty of the composition of manganin.

To obtain equality of temperature the method previously described ¹⁾ was modified in that the wires were placed in two symmetrical holes drilled in the same steel block which was placed in a thermostat. Connection was made from one of these holes to the pressure apparatus. A diagram of the pressure vessel is shown in Figure 1.

As the temperature coefficients of the wires differed only very slightly, the effect of small fluctuations in temperature was eliminated and the accuracy of the measurements thereby increased. The absolute values of these coefficients are not obtained, but, on the other hand, the value of the difference between them is, and this latter alone can affect the accuracy of the method for purposes of pressure measurement.

The manner of suspending the wires was changed in order to be more certain that they were free from strain. They were not wound on glass

¹⁾ 15th, 17th and 18th Communication of the VAN DER WAALS-Fund. These Proc. 29, 1106, 1926; 30, 47, 1927; 31, 50, 1927.

²⁾ P. W. BRIDGMAN, Proc. Am. Acc. Arts and Sc. 47, 335, 1911.

but in rectangular grooves, 0.9 mm deep and 0.8 mm wide, cut on porcelain tubes in which the 0.2 mm wires could lie without strain. Thin glass tubes

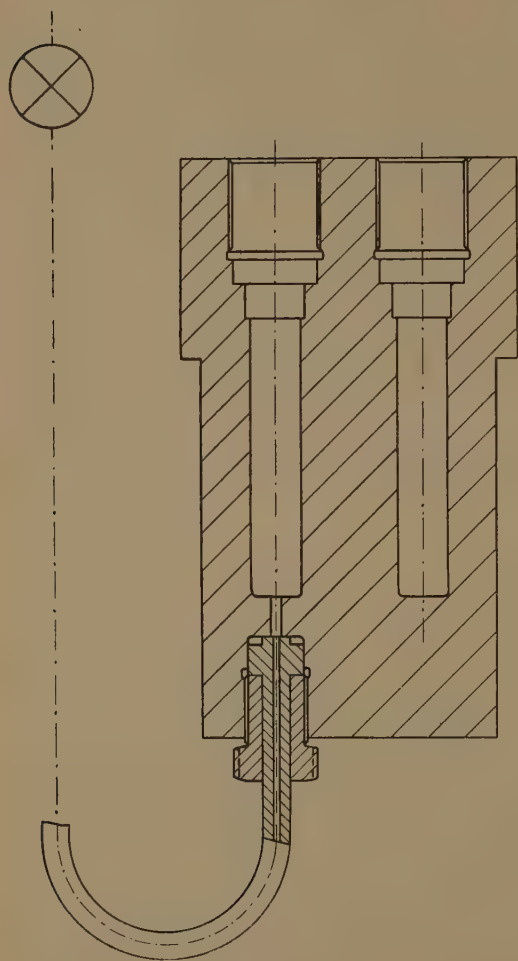


Fig. 1.

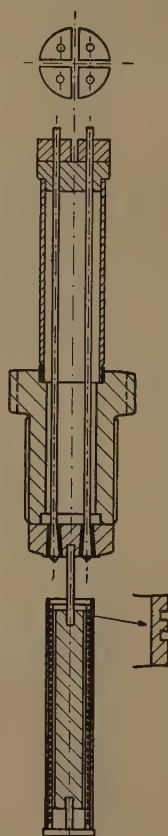


Fig. 2.

were slipped over the porcelain tubes to prevent the wires from slipping out of the grooves. The suspension of the wires is shown in Fig. 2.

"Orca" ¹⁾ was used as the insulating material for the leads through the steel vessel.

¹⁾ We are greatly indebted for this to the late Prof. MOIREU of Paris.

The electrical circuit used was the same as that described in Communication N^o. 18 (loc. cit.).

Method.

The wire to be subjected to pressure was of hard drawn manganin while the other was first heated at ca. 140° C. for one hour. The ratio of the resistance of the wires, and, therefore, the resistance of the first one, was found to decrease slowly with time, the rate of this decrease being greater at higher temperatures. In order to make correction for this effect the rate was determined before or after each set of measurements. (See Table I.)

TABLE I.

Change per hr. of Ratio of Resistance of wire under pressure to that of comparison wire $\times 10^7$.

	70° C.	51° C.	41° C.	31° C.	20° C.
					— 3 ⁵
				— 17 ⁵	— 2 ⁵
			— 70 ⁵	— 16 ⁵	— 1 ⁵
		— 445	— 47	— 10	— 1
	— 1592	— 84	— 31 ⁵	— 7	— 1 ⁵
After heating at 100° C.	— 51	— 6	— 4	— 1	—
After heating at 148° C.	— 5	—	+ 3	—	—

Measurements were first made at a temperature of ca. 20° C. and at pressures increasing by about 200 atms up to ± 1000 atms and then decreasing by the same amounts back to 1 atm.

The wires were then heated at ca. 30° C. for some time and measurements made at this temperature in a similar way. The measurements at 20° C. were then repeated and were found to give somewhat different results.

The following scheme of measurements was therefore carried out:

Wires heated to ca. 40°, Measurements made at ca. 40, 30, 20°
 50° 50, 40, 30, 20°
 70° 70, 50, 40, 30, 20°
 100° 70, 50, 40, 30, 20°
 148° 70, 50, 40, 30, 20°

(In the last case the wire to be put under pressure was removed and this alone was heated.)

Results and Conclusions.

Table II shows the values of the ratio at the various pressures obtained at 20° C. and at the highest temperatures after different heat treatments. It

TABLE II. Ratio of Resistance of wire under pressure to that of comparison wire.

Pressure in Kg/cm ²	Time h m	Pressure increasing	Time h m	Pressure decreasing	Time h m	Pressure increasing	Time h m	Pressure decreasing
19°.9 C.								
1.0					0.00	0.979 9994	4.30	0.979 9988
189.1					0.45	0.980 4292	4.05	0.980 4281
419.6					1.15	0.980 9526	3.40	0.980 9520
650.3					1.35	0.981 4767	3.15	0.981 4765
881.1					2.00	0.982 0003	2.50	0.982 0003
1062.8					2.20	0.982 4126		
31°.0 C.								
1.0	0.00	0.979 9179	3.15	0.979 9135	0.00	0.979 8833	3.20	0.979 8829
189.1	0.20	0.980 3468	2.55	0.980 3423	0.20	0.980 3114	2.55	0.980 3113
419.6	0.40	0.980 8701	2.35	0.980 8675	0.40	0.980 8353	2.35	0.980 8350
650.3	1.05	0.981 3951	2.20	0.981 3932	1.00	0.981 3591	2.15	0.981 3590
881.1	1.25	0.981 9189	2.00	0.981 9170	1.15	0.981 8828	1.55	0.981 8828
1062.8	1.40	0.982 3313			1.35	0.982 2950		
40°.8 C.								
1.0	0.00	0.979 8223	3.50	0.979 7957	0.00	0.979 6917	3.15	0.979 6913
189.1	0.30	0.980 2476	3.25	0.980 2275	0.20	0.980 1192	2.55	0.980 1191
419.6	1.20	0.980 7667	3.05	0.980 7559	0.40	0.980 6431	2.35	0.980 6427
650.3	1.35	0.981 2913	2.45	0.981 2828	1.00	0.981 1664	2.15	0.981 1666
881.1	1.50	0.981 8141	2.25	0.981 8107	1.20	0.981 6904	1.55	0.981 6898
1062.8	2.05	0.982 2259			1.35	0.982 1020		
51°.5 C.								
1.0	0.00	0.979 5121	3.19 ⁵	0.979 4318	0.00	0.979 1610	4.15	0.979 1612
189.1	0.19	0.979 9319	2.58 ⁵	0.979 8700	0.30	0.979 5881	3.50	0.979 5885
419.6	0.38	0.980 4500	2.38 ⁵	0.980 4036	1.30	0.980 1116	3.30	0.980 1116
650.3	1.01	0.980 9664	2.21	0.980 9355	1.50	0.980 6349	3.10	0.980 6349
881.1	1.19	0.981 4849	1.58	0.981 4694	2.10	0.981 1582	2.50	0.981 1579
1062.8	1.38	0.981 8909			2.30	0.981 5695		

TABLE II (Continued).

Pressure in Kg/cm ²	Time h m	Pressure increasing	Time h m	Pressure decreasing	Time h m	Pressure increasing	Time h m	Pressure decreasing
		70°.0 C.				19°.9 C.		
1.0	0.00	0.978 9081	3.20 ⁵	0.978 4045	0.00	0.977 8046	3.25	0.977 8048
189.1	0.20 ⁵	0.979 2735	3.00 ⁵	0.978 8783	0.30	0.978 2301	3.05	0.978 2307
419.6	0.46	0.979 7274	2.42	0.979 4467	0.50	0.978 7518	2.45	0.978 7520
650.3	0.59 ⁵	0.980 2156	2.20 ⁵	0.980 0218	1.05	0.979 2742	2.25	0.979 2734
881.1	1.21	0.980 6861	4.01 ⁵	0.989 5918	1.25	0.979 7952	2.05	0.979 7951
1062.8	1.41	0.981 0523			1.45	0.980 2062		
		70°.0 C.				19°.9 C.		
1.0	0.00	0.972 8144	3.01	0.972 8036	0.00	0.972 7522	3.20	0.972 7524
189.1	0.22	0.973 2369	2.41	0.973 2291	0.20	0.973 1740	3.00	0.973 1739
419.6	0.37	0.973 7573	2.22	0.973 7503	0.40	0.973 6908	2.40	0.973 6911
650.3	0.55	0.974 2764	2.04	0.974 2717	1.00	0.974 2075	2.20	0.974 2077
881.1	1.14	0.974 7957	1.48	0.974 7933	1.20	0.974 7243	2.00	0.974 7246
1062.8	1.31	0.975 2045			1.40	0.975 1316		
		70°.0 C.				19°.9 C.		
1.0	0.00	0.965 3366	3.20	0.965 3355	0.00	0.965 3609	3.20	0.965 3608
189.1	0.20	0.965 7559	3.00	0.965 7548	0.20	0.965 7772	3.00	0.965 7777
419.6	0.50	0.966 2695	2.40	0.966 2688	0.40	0.966 2884	2.40	0.966 2883
650.3	1.05	0.966 7834	2.20	0.966 7830	1.00	0.966 7992	2.20	0.966 7992
881.1	1.25	0.967 2970	2.00	0.967 2971	1.20	0.967 3097	2.00	0.967 3098
1062.8	1.40	0.967 7016			1.40	0.967 7119		

can be seen from the figures that, after heat treatment, the variation of the resistance of manganin with pressure for lower temperatures was sufficiently reproducible to make it possible to use this effect to measure pressure. This may be more clearly seen when the figures are taken in conjunction with the values of the corresponding pressure coefficients for the different ranges of pressure. (Table III.) The accuracy of the method depends only on the constancy of the pressure coefficient over the whole pressure range and not on its actual value.

The greatest difference between the pressure coefficients, after the final heat treatment, is $^3/_{2300}$ which, over a pressure range of 200 atms.

TABLE III.
Pressure coefficient $\times 10^9$.

Pressure range in Kg/cm ²		19° 9 C.	(Continued)		
			Pressure range in Kg/cm ²	51° 5 C.	19° 9 C.
1.0 — 189.1		2331			
189.1 — 419.6		2318	650.3 — 881.1	2320	2315
419.6 — 650.3		2318	881.1 — 1062.8	2322	2313
650.3 — 881.1		2316			
881.1 — 1062.8		2315		70° 0 C.	19° 9 C.
	31° 0 C.	19° 9 C.	1.0 — 189.1	2283	2315
1.0 — 189.1	2327	2324	189.1 — 419.6	2349	2314
189.1 — 419.6	2321	2319	419.6 — 650.3	2309	2313
419.6 — 650.3	2323	2317	650.3 — 881.1	2318	2310
650.3 — 881.1	2316	2316	881.1 — 1062.8	2321	2313
881.1 — 1062.8	2321	2315		70° 0 C.	19° 9 C.
	40° 8 C.	19° 9 C.	1.0 — 189.1	2317	2305
1.0 — 189.1	2327	2321	189.1 — 419.6	2322	2305
189.1 — 419.6	2326	2319	419.6 — 650.3	2318	2302
419.6 — 650.3	2324	2316	650.3 — 881.1	2319	2303
650.3 — 881.1	2322	2316	881.1 — 1062.8	2318	2303
881.1 — 1062.8	2320	2313		70° 0 C.	19° 9 C.
	51° 5 C.	19° 9 C.	1.0 — 189.1	2310	2295
1.0 — 189.1	2324	2320	189.1 — 419.6	2309	2295
189.1 — 419.6	2327	2318	419.6 — 650.3	2308	2294
419.6 — 650.3	2328	2316	650.3 — 881.1	2307	2292
			881.1 — 1062.8	2306	2292

corresponds to an uncertainty of about ± 150 gms/cm². This difference is equivalent to a change of 5 in the last figure given for the ratio. One in the latter, however, is equivalent to $1/10000$ ohm in the resistance box which is of the order of the contact resistance at each plug, so that this last figure is uncertain. A greater accuracy for the measurement of pressure might be obtained by using a more sensitive electrical circuit.

Qualitative conclusions can also be drawn from the measurements that have been made.

As would be expected the resistance was lowered by the heat treatment (v. Table II).

It can be seen from Table III that the pressure coefficients increase with increasing temperature and therefore that the temperature coefficient is increased by increasing the pressure¹⁾.

Any pressure hysteresis that might have occurred was masked in the figures obtained by the change of the resistance with time. Table IV shows

TABLE IV.

70° C.			51° C.			41° C.			31° C.			20° C.		
A	C	A _{corr}	A	C	A _{corr}	A	C	A _{corr}	A	C	A _{corr}	A	C	A _{corr}
												6	16	10
									44	57	13	4	8	4
						266	270	4	28	51	23	3	5	2
			803	1480	677	135	171	36	19	37	18	- 2	4	6
5036	5319	283	280	285	5	113	104	- 9	- 8	23	31	- 2	5	7
108	154	46	17	20	3	16	12	- 4	6	3	- 3	- 2	0	2
11	17	6	3	0	- 3	- 13	- 12	1	0	0	0	+ 1	0	- 1

A = Difference between the observed ratios before and after application of pressure $\times 10^7$.

C = Correction for change of ratio with time $\times 10^7$.

A_{corr} = The corrected difference between the ratios $\times 10^7$.

the difference between the ratio at 1 atm before and after the application of pressure, the time correction, and the difference between the ratios corrected for time. From this it will be seen that in all cases, where the values of this final difference are greater than the experimental accuracy, they have the same sign. While this cannot be taken as a conclusive proof, it is an indication that hysteresis did occur before sufficient heat treatment had been given.

Finally from inspection of Table III and similar results obtained at the intermediate temperatures it seems possible that the pressure coefficient varied slightly with the pressure, but that this effect also disappeared after the heat treatment of the wire.

¹⁾ This follows from the fact that $\frac{\partial \beta}{\partial T} = \frac{\partial \alpha}{\partial p}$ where $\beta = \frac{\partial R}{\partial p}$ and $\alpha = \frac{\partial R}{\partial T}$.

Mathematics. — *Recherche sur quelques phénomènes d'interférence des courbes de vibration.* Par J. W. N. LE HEUX. (Communicated by Prof. HENDRIK DE VRIES).

(Communicated at the meeting of December 21, 1929).

1. *Composition de quatre mouvements vibratoires, dirigés dans deux sens rectangulaires.*

Soient OX et OY les deux directions principales d'une lame cristalline. La vibration rectiligne

$$s = a \cos 2\pi \frac{t}{T}$$

d'un rayon de lumière polarisée, pénétrant normalement dans le cristal et faisant avec l'axe des X un angle α , se dédouble dans les deux vibrations

$$x = a \cos \alpha \cos 2\pi \frac{t}{T} \quad y = a \sin \alpha \cos 2\pi \frac{t}{T}.$$

Si d est la longueur de la route, commune à ces deux vibrations et posant

$$\frac{V}{V_x} = n_1 \quad \frac{V}{V_y} = n_2$$

on a

$$x = a \cos \alpha \cos 2\pi \left(\frac{t}{T} - \frac{dn_1}{\lambda} \right)$$

$$y = a \sin \alpha \cos 2\pi \left(\frac{t}{T} - \frac{dn_2}{\lambda} \right)$$

Soit la différence de phase $\frac{2\pi d}{\lambda} (n_2 - n_1) = \Delta$ et posons

$$2\pi \left(\frac{t}{T} - \frac{dn_1}{\lambda} \right) = \varphi$$

les deux mouvements rectangulaires deviennent

$$x = a \cos \alpha \cos \varphi \quad y = a \sin \alpha \cos (\varphi + \Delta), \quad . \quad . \quad . \quad . \quad . \quad . \quad (I)$$

Pour α constant, la vibration émergente est elliptique et cette ellipse prend une des formes d'un unisson de Lissajous. Pour α variable (rayons polarisés convergents) on obtient une courbe, que l'on peut considérer comme un système d'ellipses, si l'on admet pour α une variation par intervalles égaux. Ces ellipses Δ sont inscrites dans les rectangles sur les doubles coordonnées des points a du cercle $r = a$.

Prenons au lieu de ce cercle l'ellipse

$$x = a \cos \alpha \quad y = a \cos (\alpha + \theta) \quad (\theta \text{ constant})$$

La formule généralisée devient alors

$$x = a \cos \alpha \cos \varphi \quad y = a \cos (\alpha + \theta) \cos (\varphi + \Delta) \quad \dots \quad (\text{II})$$

comprenant toutes les ellipses invariables Δ , inscrites dans les rectangles sur les doubles coordonnées des points de l'ellipse invariable θ .

Cette remarque nous donne le moyen de tracer un système à l'aide de la règle et du compas.

La formule (II) peut être écrite :

$$x = \frac{1}{2} a \{ \cos (\varphi + \alpha) + \cos (\varphi - \alpha) \}$$

$$y = \frac{1}{2} a [\cos \{ (\varphi + \alpha) + (\Delta + \theta) \} + \cos \{ (\varphi - \alpha) + (\Delta - \theta) \}]$$

par conséquent : les mêmes courbes de vibration résultent de la composition de quatre mouvements vibratoires.

Cette remarque nous donne l'idée de construire un appareil, qui trace très rapidement et surtout très exactement nos figures compliquées.

Lorsqu'un tel système contient beaucoup d'ellipses (p.e. 100) on peut observer des courbes d'interférence, qui ressemblent fort aux images, que l'on voit en examinant une lame cristalline réfringente ou biréfringente en lumière polarisée et convergente. L'Etude de ces courbes d'interférence est le but du présent travail.

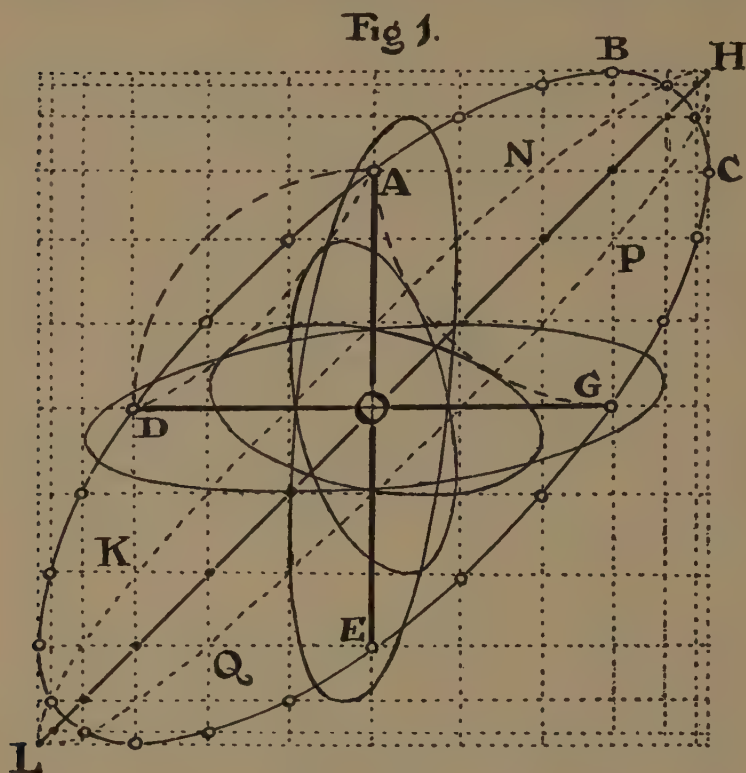
2. Tracé graphique des courbes de vibration par la règle et le compas.

Fig. 1 donne un exemple de la construction d'un système. d'Abord, on a tracé l'ellipse invariable θ , c'est la courbe $D A B C G E$, qui procurera les rectangles circonscrits des ellipses Δ , dont on voit six exemplaires, les deux directions principales $A E$ et $D G$ y comprises. ($A E = D G = 2a \cos \theta$). Par ces deux directions, l'ellipse est divisée en quatre pièces. Les pièces diamétrales donnent les mêmes rectangles. Lorsque le quadrant du cercle est divisé en six (n) parts égales (comme pour la construction de l'ellipse θ), cette ellipse est tracée par 24 ($4n$) points et donne ainsi 12 ($2n$) rectangles, par conséquence une figure, composée de 12 ($2n$) ellipses (dont 2 au moins sont des lignes droites). La division du quadrant en 6 (n) parts égales donne pour a un accroissement par intervalles de $\frac{\pi}{12} \left(\frac{\pi}{2n} \right)$.

L'Examen de la fig. 1 fait voir, que la figure totale sera composée de deux parties: 1° l'ensemble des ellipses engendrées par l'arc $A D$ et 2° l'ensemble des ellipses engendrées par l'arc $A B C G$.

Parceque l'arc $A D$ est situé entre la droite $A D$ et le segment de cercle $A D$, l'enveloppe pour $\Delta = \frac{\pi}{2}$ sera située entre l'astéroïde sur les directions principales et le carré sur la droite $A D$. C'est cette enveloppe







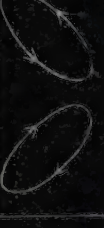

en forme de carré à côtés concaves (vers l'origine) et avec les directions principales comme diagonales, que l'on voit paraître, aussitôt qu'une



différence θ de phase se montre. Le carré croît en grandeur avec θ et est devenu un carré pur pour $\theta = \frac{\pi}{2}$.

Pour des valeurs de Δ autres que $\frac{\pi}{2}$, on voit un phénomène analogue : le carré devient un rectangle à quatre côtés concaves (voir la courbe pointillée DAG fig. 1) ou à deux côtés concaves et deux côtés convexes. Ce dernier cas est évident, parceque pour $\Delta = 0$, la figure devient un réseau, ayant pour enveloppe extérieure l'ellipse θ et pour enveloppe intérieure l'origine. l'Arc elliptique $ABCG$ se compose des parties AB , BC et CG , dont seule la partie BC contribue à engendrer l'enveloppe. Le rayon de courbure pour cette partie est plus petit que le rayon de cercle $OB = OC$, donc l'enveloppe sera un peu convexe vers l'origine.



	A	B	C	D	E
$x = +\cos(p+\alpha) + \cos(p-\alpha)$ $y = +\cos(p+\alpha) + \cos(p-\alpha)$ 1	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $+ \cos(p+\alpha) + \cos(p-\alpha)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $+ \cos(p+\alpha/5) + \cos(p-\alpha/5)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $+ \cos(p+\alpha/60) + \cos(p-\alpha/60)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \sin(p+\alpha) - \sin(p-\alpha)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \sin(p+\alpha) - \sin(p-\alpha)$ 
$x = +\cos(p+\alpha) + \cos(p-\alpha)$ $y = +\cos(p+\alpha/30) + \cos(p-\alpha/30)$ 2	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $+ \cos(p+\alpha/30) + \cos(p-\alpha/30)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $+ \cos(p+\alpha/60) + \cos(p-\alpha/60)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $+ \cos(p+\alpha/60) + \cos(p-\alpha/60)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \sin(p+\alpha) - \sin(p-\alpha)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \cos(p+\alpha/75) + \cos(p-\alpha/75)$ 
$x = +\cos(p+\alpha) + \cos(p-\alpha)$ $y = +\cos(p+\alpha/45) + \cos(p-\alpha/45)$ 3	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $+ \cos(p+\alpha/60) + \cos(p-\alpha/60)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \sin(p+\alpha) - \sin(p-\alpha)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \cos(p+\alpha/30) + \cos(p-\alpha/30)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \cos(p+\alpha/60) + \cos(p-\alpha/60)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \cos(p+\alpha/45) + \cos(p-\alpha/45)$ 
$x = +\cos(p+\alpha) + \cos(p-\alpha)$ $y = +\cos(p+\alpha/75) + \cos(p-\alpha/75)$ 4	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \sin(p+\alpha) - \sin(p-\alpha)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \cos(p+\alpha/60) + \cos(p-\alpha/60)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \cos(p+\alpha/30) + \cos(p-\alpha/30)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \cos(p+\alpha/15) + \cos(p-\alpha/15)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \cos(p+\alpha/15) + \cos(p-\alpha/15)$ 
$x = +\cos(p+\alpha) + \cos(p-\alpha)$ $y = -\sin(p+\alpha) + \sin(p-\alpha)$ 5	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \sin(p+\alpha) + \sin(p-\alpha)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \cos(p+\alpha/75) + \cos(p-\alpha/75)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \cos(p+\alpha/15) + \cos(p-\alpha/15)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \cos(p+\alpha/15) + \cos(p-\alpha/15)$ 	$+ \cos(p+\alpha) + \cos(p-\alpha)$ $- \cos(p+\alpha) + \cos(p-\alpha)$ 

Comme partie extérieure de l'enveloppe on a une espèce de carré à coins arrondis, mais toujours convexe vers O. Pour les valeurs de Δ autres que $\frac{\pi}{2}$, le carré devient un rectangle, dont deux côtés peuvent être concaves.

Toutes ces particularités se présentent dans la table I, dont les 25 figures ont été construites à la manière indiquée. A l'aide de la formule

$$\begin{aligned}x &= a \cos \alpha \cos \varphi \\y &= \cos (\alpha + \theta) \cos (\varphi + \Delta)\end{aligned}$$

on a :

Pour les figures d'une même colonne, Δ est constant, pendant que θ prend les valeurs successives 0, $\frac{\pi}{12}$, $\frac{3\pi}{12}$, $\frac{5\pi}{12}$ et $\frac{6\pi}{12}$. (de haut en bas).

Pour les figures d'une même ligne, θ est constant, pendant que Δ prend les valeurs successives 0, $\frac{\pi}{12}$, $\frac{3\pi}{12}$, $\frac{5\pi}{12}$ et $\frac{6\pi}{12}$. (de gauche à droite).

Cependant, on n'a pas dessiné, comment la courbe passe par toutes ses apparences — nous y revenons bientôt.

La table II est composée pour la formule

$$\begin{aligned}x &= \frac{1}{2} a \{ \cos (\varphi + \alpha) + \cos (\varphi - \alpha) \} \\y &= \frac{1}{2} a [\cos \{ (\varphi + \alpha) + (\Delta + \theta) \} + \cos \{ (\varphi - \alpha) + (\Delta - \theta) \}]\end{aligned}$$

en donnant à Δ et à θ successivement les cinq valeurs de la table I.

Elle nous apprend, pour une figure donnée de la table I :

1°. quels sont les quatre mouvements vibratoires, d'où elle résulte ;

2°. les valeurs de θ et de Δ , en prenant le chiffre initial de la différence de phase dans la ligne et dans la colonne, auxquelles appartient cette figure.

Ainsi, le cas 2D donne pour θ , 15° et pour Δ , 75° .

3. *Tracé graphique des courbes de vibration à l'aide d'un appareil.*

Quatre pendules, dont les mouvements sont transportés par des règles d'aluminium, sont groupés en forme de croix orthogonale de manière que deux des mouvements (1 et 2) font tracer une figure de Lissajous sur un petit tableau d'aluminium, dont le centre se trouve au-dessus du point de rencontre des pendules 3 et 4 et dont chaque point décrit ainsi une figure de Lissajous lui-même.

Dans notre cas, les pendules 1 et 2 doivent être exactement égaux, pour que la composition de leurs mouvements donne l'ellipse invariable.

$$\begin{aligned}x &= \frac{1}{2} a \cos (\varphi + \alpha) \\y &= \frac{1}{2} a \cos \{ (\varphi + \alpha) + (\Delta + \theta) \}.\end{aligned}$$

Cette courbe est tracée par le point fin d'un petit tuyau capillaire en encre noire sur une feuille de papier glacé. La différence constante de

phase, qui détermine la forme de l'ellipse initiale, est obtenue en faisant commencer un des pendules quelque temps après l'autre.

De même, les pendules 3 et 4 doivent être exactement égaux, pour que le tableau avec le papier décrive l'ellipse

$$\begin{aligned}x &= \frac{1}{2} a \cos (\varphi - \alpha) \\y &= \frac{1}{2} a \cos \{(\varphi - \alpha) + (\Delta - \theta)\}\end{aligned}$$

dont la forme dépend de la différence constante de phase. Les pendules 1 et 3 diffèrent un peu en longueur — les pendules 2 et 4 montrent exactement la même différence. En exprimant cette condition dans la formule II on voit que l'ellipse 1—3, et de même l'ellipse 2—4, montreraient, quand on les fit tracer, une différence variable de phase de 2α , par conséquence, elles passeraient par toutes leurs apparences distinctes.

Supposant, que la différence 2α accroit par intervalles de $\frac{\pi}{2n}$, la figure de quatre mouvements contient $2n$ ellipses.

Pour toutes les figures de la table I, $2n = 12$.

Dans la table III, on ne trouve que des courbes, tracées par l'appareil et diminuées en raison de 1 : 2.

4. *Cas de deux mouvements circulaires inverses.* Table I, fig. 5A.

Les figures analogues des tables I et II montrent, qu'il s'agit de l'explication, donnée par Fresnel, du pouvoir rotatoire des cristaux ou de la trajectoire d'un électron négatif dans un plan, perpendiculaire aux lignes de force d'un champ magnétique.

De

$$\begin{aligned}x &= \cos (\varphi + \alpha) + \cos (\varphi - \alpha) \\y &= -\sin (\varphi + \alpha) + \sin (\varphi - \alpha)\end{aligned}$$

on déduit

$$\begin{aligned}x &= 2 \cos \alpha \cos \varphi \\y &= -2 \sin \alpha \cos \varphi\end{aligned}$$

c'est à dire :

Si deux cercles égaux ($r=1$) sont parcourus en sens inverses et avec une différence de phase, qui croît jusqu'à 2α , la vibration rectiligne $x=2 \cos \varphi$, $y=-2 \cos \varphi$ tourne autour de l'origine d'un angle α .

Posant $2\alpha = \frac{\pi}{n}$, nous avons :

Lorsqu'un point est soumis à l'action simultanée de deux mouvements circulaires en sens inverses et dont les vitesses sont dans le rapport de $(n+1)$ à $(n-1)$, il parcourt une „étoile" à $2n$ points de rebroussement.

La figure 5A de la table I ne fait pas voir, comment la courbe passe par toutes ses variétés. Pour l'expliquer, nous considérons la courbe donnée

$$\begin{aligned}x &= \cos (\varphi + \alpha) + \cos (\varphi - \alpha) \\y &= \sin (\varphi + \alpha) - \sin (\varphi - \alpha)\end{aligned}$$

comme la limite de la courbe

$$\begin{aligned}x &= q \cos p\varphi + p \cos q\varphi \\y &= q \sin p\varphi - p \sin q\varphi \quad p < q\end{aligned}$$

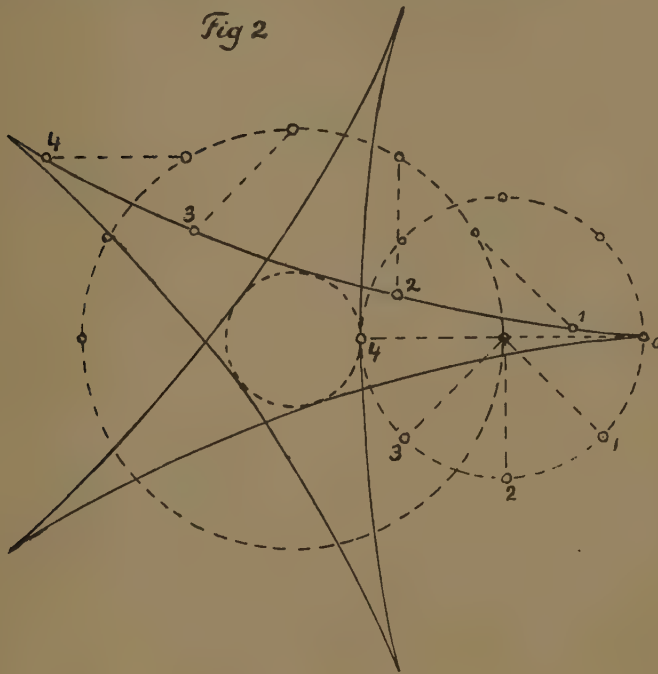
quand p se rapproche de q ($= 1$).

Cette dernière courbe est l'hypocycloïde, parcourue par un point du cercle de rayon p , qui roule à l'intérieur d'un cercle de rayon $p + q$.

Fig. 2 montre la construction de la courbe.

$$\begin{aligned}x &= 3 \cos 2\varphi + 2 \cos 3\varphi \\y &= 3 \sin 2\varphi - 2 \sin 3\varphi\end{aligned}$$

étoile à $2 + 3$ points de rebroussement. On déduit de cette figure que des boucles seront formées pour un rayon du cercle roulant > 2 .



Quand le rapport $\frac{p\varphi}{q\varphi}$ est à peu près égal à 1 ($= \frac{\varphi + a}{\varphi - a}$), l'étoile se compose d'un très grand nombre de rayons. Alors, les boucles se forment aisément par une petite différence de diamètre des cercles, ou, pour les figures de vibration, par une petite différence d'amplitude. Parceque les deux pendules de l'appareil, qui font mouvoir le tableau, éprouvent un

Donc, dans ce cas, "l'étoile" serait composée de $n = \pm 18000$ rayons.

5. Cas de deux mouvements circulaires égaux.

Outre le cas

$$\begin{aligned} x &= \cos(\varphi + \alpha) + \cos(\varphi - \alpha) \\ y &= -\sin(\varphi + \alpha) + \sin(\varphi - \alpha) \quad \text{(IV)} \end{aligned}$$

nous voulons examiner le cas

$$\begin{aligned} x &= \cos(\varphi + \alpha) + \cos(\varphi - \alpha) \\ y &= -\sin(\varphi + \alpha) - \sin(\varphi - \alpha) \quad \text{(V)} \end{aligned}$$

où les deux cercles sont parcourus en sens égaux et avec une différence de phase 2α (table II, fig. 1E).

Sans cette différence, IV donne un rayon polarisé rectilignement

$x = 2 \cos \varphi$, et V donne un rayon polarisé circulairement

$x = 2 \cos \varphi$, $y = -2 \sin \varphi$.

On déduit de la formule V :

$$\begin{aligned} x &= 2 \cos \alpha \cos \varphi \\ y &= -2 \cos \alpha \sin \varphi, \end{aligned}$$

par conséquence, elle représente un cercle de rayon variable $r = 2 \cos \alpha$, et ainsi, la table I donne un système de cercles concentriques.

Quand le rayon $r = 2 \cos \alpha$ va se modifier progressivement, le point décrit une spirale (table III, fig. 11 et 12) qui s'approchera du centre à mesure que la différence de phase varie de 0 à $\frac{\pi}{4}$, qui s'en écartera

ensuite, quand cette différence varie de $\frac{\pi}{4}$ à $\frac{\pi}{2}$, qui s'en rapprochera de nouveau et ainsi de suite.

l'Accroissement progressif de la différence de phase produit :

1°. pour deux mouvements circulaires inverses une rotation continue du plan de polarisation ;

2°. pour deux mouvements circulaires égaux une variation périodique de l'amplitude entre deux limites extrêmes.

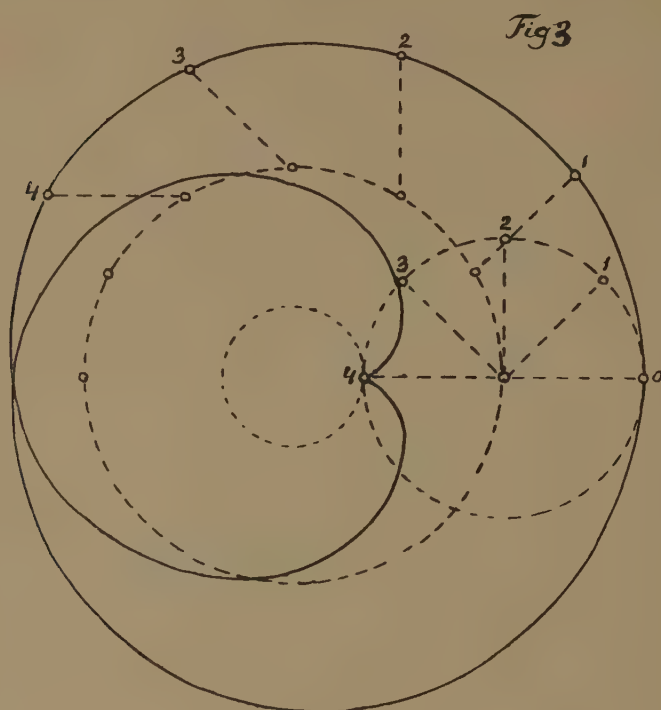
Pour nous rendre compte de quelle manière le cercle de fig. 1E passe par toutes ses variétés, nous examinons la figure

$$\begin{aligned} x &= q \cos p\varphi + p \cos q\varphi \\ y &= q \sin p\varphi + p \sin q\varphi. \end{aligned}$$

Le cas $p = 2$, $q = 3$ est construit dans fig. 3.

Pour le rapport 8 : 9, l'appareil donne fig. 13 de la table III. On peut remarquer, que l'image est composée de deux spirales, dont les points de rencontre sont situés sur un diamètre. Mais en diminuant la différence de phase à l'aide de l'appareil, on voit se présenter un phénomène, qui échappe à l'observation, quand la figure ne contient pas un très

grand nombre de courbes, savoir, que les points de rencontre vont former deux secteurs noirs (table III, fig. 14).



Examinons le cas

$$x = \cos(\varphi + a) + \cos(\varphi - a')$$

$$y = \sin(\varphi + a) + \sin(\varphi - a')$$

qui dépasse cependant la limite de nos recherches.

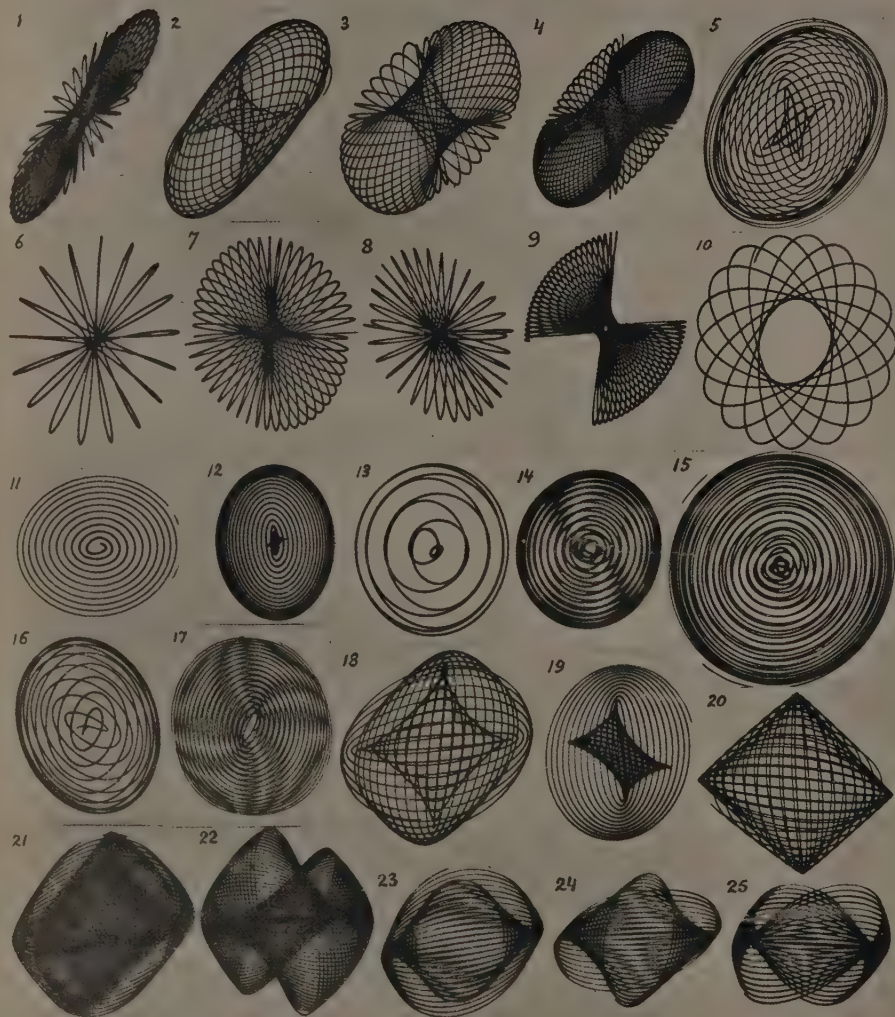
Le cercle $x = \cos(\varphi + a)$, $y = \sin(\varphi + a)$ reste fixe; la courbe $x = \cos(\varphi - a')$, $y = \sin(\varphi - a')$ passe par toutes les formes d'un unisson de Lissajous. En écrivant:

$$x = \cos \frac{a + a'}{2} \cos \left(\varphi + \frac{a - a'}{2} \right)$$

$$y = \cos a \sin \varphi$$

on voit de la construction, que le lieu des points d'intersection devient une courbe.

Cette observation est affirmée par l'expérience (fig. 15); pour une petite valeur de a , l'image ressemblera à une spirale d'Airy à deux branches.



La moindre inexactitude dans les pendules de l'appareil changera des figures comme 5 *A* (table I) en figures comme 4 *B* et 5 *B* (fig. 8 et fig. 7 de la table III). De même, fig. 1 *E* passera bientôt par des formes comme 2 *D* et 2 *E* (fig. 16 et fig. 5 de la table III). En examinant ces figures, on voit encore les deux spirales, mais avec l'apparence de deux systèmes d'ellipses concentriques, qui se coupent sous un angle quelconque.

Pour expliquer ce phénomène, nous remarquons (fig. 1), que les rectangles circonscrits des ellipses \triangle proviennent de la droite *HL* pour fig. 1 *E*, mais de l'ellipse mince *KNPQ* pour les figures 2 *E* et 2 *D*. On pourrait dire, que la droite *HL* s'est dédoublée dans les deux droites *KN* et *QP* qui, chacune d'elles, fournissent une série de rectangles, de manière que la deuxième série se déduit de la première en échangeant les côtés comme le montre fig. 1.

Pour $\triangle = \frac{\pi}{2}$, on obtient deux séries de rectangles "conjugués" dont les axes correspondants sont perpendiculaires.

Maintenant, pour un très grand nombre d'ellipses, le lieu des points d'intersection se compose de quatre secteurs noirs ou d'une croix noire à quatre branches avec une petite astéroïde noire autour du centre. L'Image totale ressemble fort à la figure d'interférence d'un cristal à un axe, traversé par un rayon polarisé de lumière convergente, perpendiculaire à l'axe, surtout en observant qu'il est possible de faire approcher les ellipses à des cercles (pour une petite valeur de θ) et que la distance des ellipses (et leur nombre) dépend de la différence de phase $2a$, que l'on donne aux pendules.

Une idée assez nette des quatre secteurs noirs donne la fig. 17 de la table III; en outre, on y voit la possibilité d'obtenir une spirale d'Airy à quatre branches. Fig. 12 de la table III montre la naissance de la deuxième spirale après l'achèvement de la première.

6. Cas de deux mouvements lineaires et rectangulaires.

Examinons, dans la table I, la dernière colonne et la dernière ligne. Commenant par la fig. 1 *E*, on obtient la figure 5 *E* par un accroissement de la différence de phase θ ; commençant par 5 *A*, on obtient la même figure 5 *E* par un accroissement de la différence de phase \triangle .

Cette figure 5 *E* est l'unisson de Lissajous, mais résultant de quatre mouvements vibratoires au lieu de deux.

Réciproquement, on peut observer un phénomène remarquable de l'unisson, résultant de la moindre différence de phase ou du fait, que les deux mouvements composants rectilignes se transforment en mouvements elliptiques.

Alors, l'unisson se dédouble et par la superposition des deux parties, on voit paraître un système d'hyperboles d'interférence.

Cette singularité, échappant à l'observation quand il s'agit d'une figure construite de la table I, même quand celle-ci renferme beaucoup d'ellip-

ses, se présente dans les figures, tracées par l'appareil (voir fig. 21, 22 et 25 de la table III).

Dans le cas d'une différence de phase Δ (sens inverses fig. 5 D) on voit paraître un système d'hyperboles à centre noir. Dans le cas d'une différence de phase θ (sens égaux, fig. 4 E) le centre est blanc.

Le dédoublement de l'unisson se produit d'une manière différente dans les deux cas.

La fig. 4 E montre un unisson à enveloppe concave dans un unisson à enveloppe convexe (Table III fig. 19, 18 et 20). La fig. 5 D est composée de deux unissons convex — concaves entrecroisés (Table III fig. 22 et 25 — ces figures sont restées inachevées pour montrer mieux les hyperboles).

Dans ces deux cas, les deux parties de la figure se rencontrent dans les quatre extrémités des directions principales.

De la figure 4 E à la figure 5 D, on passe par la figure 4 D, où l'unisson concave intérieur passe par deux côtés du carré convexe pour devenir partiellement convexe.

Un tel passage dans les figures, tracées par l'appareil, se montre dans la table III, fig. 23, 24 et 25.

Ces figures ne sont pas achevées pour montrer le changement des courbes d'interférence.

Fig. 21 donne une figure 4 E complète, sauf quelques inexactitudes. Elle montre quelques hyperboles, que la fig. 20 ne fit pas encore observer à cause d'un trop petit nombre d'ellipses.

7. Cas de deux mouvements elliptiques inégaux.

L'Examen de la table III fait voir, que dans toutes les figures, excepté celles de la première ligne, de la première colonne et fig. 5 E, un astéroïde se formera autour du centre, renfermant un système d'hyperboles.

Lorsque cet astéroïde est petit (pour une petite valeur de θ ou de Δ) il faut un très grand nombre d'ellipses pour former les courbes d'interférence. Pour les figures, dessinées de la main, dans la table I, il n'y a même pas une seule ellipse dans les cas 2 C, 2 D et 2 E pour former l'enveloppe intérieure.

Quant aux figures de la table III, tracées par l'appareil, les cas 2 et 3 ne donnent pas encore des hyperboles — celles-ci commencent à être visibles dans la fig. 4, qui contient un grand nombre d'ellipses (partiellement tracées). Cependant, le nombre est limité par l'épaisseur des lignes d'encre. Ainsi, il faut conclure par raisonnement à la possibilité de l'existence d'ovales (lemniscates), composés par les hyperboles (dont les asymptotes ont la direction des franges noires) et les courbes d'interférence comme celles de la fig. 9. Dans ce cas, les directions principales ne devraient pas être rectangulaires (fig. 8 de la table III) de manière que les courbes d'interférence seraient allongées dans deux des quadrants et comprimées dans les deux autres.

8. Résumé.

Un rayon polarisé rectilignement peut être séparé en deux rayons elliptiques, conjugués et inverses (Airy 1830) (table II 5^{ième} ligne).

Un rayon polarisé circulairement peut être séparé en deux rayons elliptiques, conjugués et égaux. (table II colonne E).

Lorsque les deux ellipses sont parcourues avec une petite différence de phase, la figure résultante montre (outre les cas de limite) autour du centre un astéroïde plus ou moins grand, qui renferme un système d'hyperboles. Ce système a le centre noir pour les mouvements inverses, le centre blanc pour les mouvements égaux.

Dans certains cas, des courbes d'interférence en forme d'ovales se montrent autour de l'astéroïde — elles semblent former, avec les hyperboles, un système de lemniscates.

Une croix noire traversant toute la figure (système d'isogyres), peut être formée en deux manières :

1^o. par des éléments de tangente horizontaux (verticaux) à l'extrémité d'un axe vertical (horizontal). Fig. 2E, cas de deux mouvements égaux à peu près circulaires.

2^o. par des éléments de tangente horizontaux (verticaux) à l'extrémité d'un axe horizontal (vertical). Fig. 5B, cas de deux mouvements inverses à peu près circulaires.

Deux secteurs noirs se forment par deux mouvements circulaires égaux.

Un espace vide autour du centre indique une différence d'amplitude. ($r = a - b$).

Une spirale d'interférence se forme, lorsqu'une des ellipses composantes reste fixe, pendant que l'autre passe par les formes d'un unisson de Lissajous.

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Mathematics. — *Anzahlbestimmung der Quadrisekanten einer algebraischen Raumkurve, die mehrfache Punkte besitzt.* Von G. SCHAAKE. (Communicated by Prof. JAN DE VRIES).

(Communicated at the meeting of December 21, 1929).

§ 1. In meiner Mitteilung: "*On the Surface of the Trisecants of a Twisted Curve which has Multiple Points*"¹⁾ habe ich bewiesen, dass die Ordnung der Trisekantenfläche einer Raumkurve k n^{ter} Ordnung mit h scheinbaren Doppelpunkten, v_3 dreifachen, v_4 vierfachen, ... v_m m -fachen Punkten angegeben wird durch die folgende Formel:

$$(n-2) \left\{ h - \frac{1}{2} n(n-1) \right\} + \sum_{i=3}^{i=m} \frac{1}{6} v_i i(i-1)(i-2).$$

Wir wollen nun untersuchen, wieviel Quadrisekanten eine derartige Raumkurve besitzt. Hierzu gebrauchen wir die Methode, die von ZEUTHEN in seinem "*Lehrbuch der Abzählenden Methoden*" (139) angegeben worden ist.

§ 2. Wir denken uns eine Raumkurve k n^{ter} Ordnung mit h scheinbaren Doppelpunkten, die einen m -fachen Punkt M besitzt. Für diese Kurve ist die Ordnung der Trisekantenfläche Δ :

$$t_3 = (n-2) \left\{ h - \frac{1}{2} n(n-1) \right\} + \frac{1}{6} m(m-1)(m-2).$$

Überdies denken wir uns zwei nicht auf k gelegene Punkte A_1 und A_2 und eine Erzeugende t von Δ . Die Ebenen $A_1 t$ und $A_2 t$ schneiden k ausserhalb t bzw. in $n-3$ Punkten P_1 und $n-3$ Punkten P_2 . Wenn wir für jede Erzeugende t von Δ den daraus abgeleiteten Punkten P_1 die daraus abgeleiteten Punkte P_2 zuordnen, so bekommen wir eine Verwandtschaft (P_1, P_2) auf k , auf welche wir die CAYLEY-BRILL'sche Formel:

$$\epsilon = a_1 + a_2 + 2p k_{12}$$

anwenden werden.

Für a_1 , das ist die Zahl der Punkte P_1 , die einem gegebenen Punkte P_2 zugeordnet sind, finden wir, weil die Kurve k eine $(h-n+2)$ -fache Kurve von Δ ist:

$$(t_3 - h + n - 2)(n - 3);$$

auch finden wir, dass $a_2 = a_1$ ist.

Wenn wir die Punkte, die eine Gerade t von Δ mit der Kurve k

¹⁾ These Proceedings, 27, 362.

gemein hat, einander zuordnen, so ergibt sich eine Verwandtschaft (P_3, P_4) . Aus der wohlbekannten Formel:

$$\varepsilon = p + q - g$$

folgt, dass diese

$$n(n-2)(n-5) - 2h(n-6) - m(m-1)(m-2)$$

Koinzidenzen besitzt.

Von M gehen

$$h - mn + m(m+1)$$

Trisekanten aus, die k^n zweimal ausserhalb M schneiden und m -fache Erzeugenden von \triangle sind. Weiter liegen in jeder der $\frac{1}{2}m(m-1)$ gemeinsamen Berührungsebenen von zwei Zweigen noch

$$n - m - 2$$

von M ausgehende Trisekanten, die k^n nur einmal ausserhalb M schneiden.

Der Punkt M ist also ein

$$m \{ h - \frac{1}{2}(n-m)(m+1) + 1 \} -$$

facher Punkt von \triangle .

Die letztgenannte Verwandtschaft besitzt

$$\frac{1}{2}m(m-1)(n-m-2)$$

in M liegende Koinzidenzen, die doppelt gezählt werden müssen, wie man mit Hilfe der bekannten Regel von ZEUTHEN leicht findet. Es gibt also

$$n(n-2)(n-5) - 2h(n-6) - m(m-1)(n-4)$$

Koinzidenzen (P_3, P_4) , die aus aufeinanderfolgenden Punkten der Kurve bestehen.

Nach dem CAYLEY-BRILL'schen Satze ist diese Zahl gleich:

$$4(h-n+2) + 2k_{34}p$$

wenn k_{34} die Wertigkeit der Verwandtschaft (P_3, P_4) und $2p = (n-1)(n-2) - 2h - m(m-1)$ ist.

Hieraus folgt:

$$k_{34} = n - 4.$$

Weil die einem Punkte P_3 zugeordneten Punkte P_1 zusammen mit den P_3 zugeordneten Punkten P_4 von $h - n + 2$ P_3 enthaltenden Ebenen ausgeschnitten werden, ist:

$$h - n + 2 = k_{13} + k_{34}$$

und weil die einem Punkte P_1 zugeordneten Punkte P_2 zusammen mit den P_1 zugeordneten Punkten P_3 von $t_3 - h + n - 2$ nicht durch P_1 gehenden Ebenen ausgeschnitten werden, ist:

$$0 = k_{12} + k_{13}.$$

Für die Wertigkeit der Verwandtschaft (P_1, P_2) findet man so:

$$k_{12} = 2(n-3) - h.$$

Die Koinzidenzen dieser Verwandtschaft liegen

1°. In den $n-3$ Schnittpunkten von k mit den Ebenen durch $A_1 A_2$ und den $A_1 A_2$ schneidenden Trisekanten, welche nicht zu diesen Trisekanten gehören;

2°. In den $4t_4$ Schnittpunkten von k mit den nicht durch M gehenden Quadrisekanten;

3°. In M . Wenn P_1 auf einem Zweige in M kommt, gehen

$$(m-1)\{h-mn+m(m+1)\} + \frac{1}{2}(m-1)(m-2)(n-m-2)$$

der zugehörigen Geraden $P_3 P_4$ schliesslich durch M . Die Verwandtschaft (P_1, P_2) hat eben so viele Koinzidenzen auf dem angenommenen Zweige in M . In M fallen also im Ganzen

$$m(m-1)\{h-mn+m(m+1)\} + \frac{1}{2}m(m-1)(m-2)(n-m-2)$$

Koinzidenzen.

Die Formel:

$$\varepsilon = a_1 + a_2 + 2p k_{12}$$

gibt nun:

$$t_4 = \frac{1}{24}[12h(h-4n+11) - n(n-2)(n-3)(n-13)] + \\ + \frac{1}{24}m(m-1)(m-2)(4n-3m-15).$$

Wenn die Kurve k v_3 dreifache, v_4 vierfache, ..., v_m m -fache Punkte besitzt, so finden wir auf dieselbe Weise, dass die Kurve

$$\frac{1}{24}[12h(h-4n+11) - n(n-2)(n-3)(n-13)] + \\ + \sum_{i=3}^{i=m} \frac{1}{24} v_i i(i-1)(i-2)(4n-3i-15)$$

Quadrisekanten besitzt, die nicht durch einen der angedeuteten vielfachen Punkte gehen.

§ 3. Wir wollen das gefundene Resultat anwenden auf die Bestimmung der Zahl der kubischen Raumkurven k^3 , die durch vier gegebene Punkte P_1, \dots, P_4 gehen und eine gegebene Raumkurve k n^{ter} Ordnung mit h scheinbaren Doppelpunkten viermal schneiden. Diese Zahl ist gleich der Zahl der nicht durch einen der vielfachen Punkte gehenden Quadrisekanten der Kurve k' , die aus k kann werden abgeleitet mittelst der Transformation

$$x'_i = \frac{1}{x_i} (i=1, 2, 3, 4).$$

Die Kurve k' ist von der Ordnung $3n$, sie besitzt n -fache Punkte in den Eckpunkten des Koordinatentetraeders und sie hat $n(2n-1) + h$ scheinbare Doppelpunkte, weil k' dasselbe Geschlecht hat wie k .

Das in § 2 gefundene Resultat lehrt nun, dass es

$$\frac{1}{8} n (n-1) (n-2) (n-3) + \frac{1}{2} (4n^2 - 14n + 11) h + \frac{1}{2} h^2$$

kubische Raumkurven gibt, die durch vier gegebene Punkte gehen und eine Raumkurve n^{ter} Ordnung mit h scheinbaren Doppelpunkten viermal schneiden.

Durch Anwendung der in § 1 angegebenen Formel, und wenn wir achten auf die in § 2 abgeleitete Vielfachheit von M als Punkt der Fläche Δ , finden wir, dass die kubischen Raumkurven, die durch vier gegebene Punkte gehen und eine Raumkurve n^{ter} Ordnung mit h scheinbaren Doppelpunkten dreimal schneiden, eine Fläche bilden von der Ordnung:

$$(5n-6) [\frac{1}{2} n (n-1) + h],$$

die in jedem der gegebenen Punkte einen

$$[\frac{1}{3} n (n-1) (4n-5) + (3n-4) h] -$$

fachen Punkt besitzt und wofür die gegebene Raumkurve eine

$$[2(n-1)^2 + h] -$$

fache Kurve ist.

§ 4. Die in § 2 gefundene Formel kann auch angewendet werden auf die Bestimmung der Zahl der Kreise, die durch einen gegebenen Punkt des Raumes gehen und eine gegebene Raumkurve von der Ordnung n mit h scheinbaren Doppelpunkten viermal schneiden. Diese Zahl ist gleich der Anzahl der nicht durch den vielfachen Punkt gehenden Quadrisekanten der Kurve k' , die aus k entsteht durch eine Inversion mit Zentrum in P .

Diese Kurve ist von der Ordnung $2n$; sie hat einen n -fachen Punkt in P und sie besitzt $n(n-1) + h$ scheinbare Doppelpunkte.

Mit Hilfe der in § 2 gefundenen Formel finden wir nun, dass es:

$$\frac{1}{24} n (n-1) (n-2) (n-3) + \frac{1}{2} h (2n^2 - 10n + 11) + \frac{1}{2} h^2$$

Kreise gibt, die durch einen gegebenen Punkt gehen und eine gegebene Kurve n^{ter} Ordnung mit h scheinbaren Doppelpunkten viermal schneiden.

Auch finden wir, dass die Kreise, die durch einen gegebenen Punkt gehen und eine gegebene Kurve n^{ter} Ordnung mit h scheinbaren Doppelpunkten dreimal schneiden, eine Fläche bilden von der Ordnung

$$\frac{7}{6} n (n-1) (n-2) + (3n-4) h,$$

die im gegebenen Punkte einen

$$[\frac{5}{6} n (n-1) (n-2) + (2n-2) h] -$$

fachen Punkt besitzt und wofür die gegebene Kurve eine

$$[(n-1)(n-2) + h] -$$

fache und der absolute Kreis eine

$$\left[\frac{1}{3} n(n-1)(n-2) + (n-2)h\right] -$$

fache Kurve ist.

§ 5. Die in § 2 gefundene Formel gilt nicht, wenn der m -fache Punkt drei oder mehr durch diesen Punkt gehende Zweige besitzt, die dort dieselbe Ebene berühren.

Besitzt der m -fache Punkt nur drei Zweige, die dort dieselbe Ebene berühren, so haben wir der in § 1 zitierten Mitteilung gemäss die Anzahl t_3 um eins zu vermindern. In der Formel:

$$\varepsilon = p + q - g,$$

angewendet auf die Verwandtschaft (P_3, P_4) finden wir also nun für die Anzahl ε sechs mehr.

Wenn P_1 auf einem der genannten Zweige in M kommt, nähern zwei der durch P_1 gehenden Erzeugenden sich zu Geraden von τ ; wenn P_1 auf einem der zwei anderen Zweige in M kommt, so nähern zwei der durch P_1 gehenden Erzeugenden sich denselben Geraden von τ . Δ besitzt nun eine durch M gehende Erzeugende weniger.

Also fallen zwei Koïnzenzen hinweg; jede der beiden angegebenen in τ gelegenen Geraden vertritt nun sechs Koïnzenzen, also vier mehr. In der Formel

$$\varepsilon = a_1 + a_2 + 2k_{34}p$$

behält ε also denselben Wert; daher finden wir für k_{34} wieder dasselbe.

In der Formel

$$\varepsilon = a_1 + a_2 + 2k_{12}p$$

sind a_1 und a_2 beide um $n-3$ kleiner. Weil ε den Bestandteil $(n-3)t_3$ enthält wird diese Zahl erstens um $n-3$ kleiner.

Die Zahl der in M fallenden Koïnzenzen wird um $m-2$ kleiner durch die ausgeschiedene Erzeugende von Δ . Die beiden in τ liegenden Geraden von Δ , die oben angegeben sind und ursprünglich je $m-2$ Koïnzenzen gaben, liefern deren nun je $m-3$. Weiter geben die $n-m-3$ anderen in τ liegenden Erzeugenden von Δ , die dreifache Erzeugenden dieser Fläche sind, je auf jedem der drei τ berührenden Zweige noch eine Koïnzenz, weil eine unendlich benachbarte Erzeugende auf einem derartigen Zweige ein Punktepaar (P_1, P_2) liefert, wofür $P_1 P_2$ unendlich kein zweiter Ordnung ist.

Das Ergebnis der in § 2 gefundenen Formel muss also nun um $n-m-3$ vermindert werden. Wenn M ein dreifacher Punkt ist, so gilt also nun die Formel, welche für eine Kurve ohne mehrfache Punkte abgeleitet worden ist.

Das letztere kann ersichtlich auch gesagt werden von einer Kurve, die zu einer algebraischen Fläche gehört und mehrfache Punkte besitzt, welche in nichtsingulären Punkten dieser Fläche gelegen sind.

Mathematics. — *Ueber Differentialinvarianten einer verallgemeinerten GALILEI—NEWTON-Gruppe.* Von C. G. G. VAN HERK. (Communicated by Prof. R. WEITZENBÖCK).

(Communicated at the meeting of December 21, 1929).

I.

§ 1. Einleitung.

Die Veranlassung zu den folgenden Betrachtungen bilden gewisse physikalische Spekulationen, auf die hier nicht eingegangen wird.

Sei $(x_1 x_2 x_3 t)$ ein 4-dimensionales Raum-Zeit-Kontinuum, und sei T eine Transformation:

$$\left. \begin{array}{l} x_i = e_i + e_{i1} \bar{x}_1 + e_{i2} \bar{x}_2 + e_{i3} \bar{x}_3 \quad (i = 1, 2, 3) \\ t = \bar{t} \end{array} \right\} \quad . \quad . \quad . \quad (1)$$

wo $\|e_{ik}\|$ eine orthogonale Matrix ist; für $i \neq k$ sind die e_{ik} bis auf eine Zweideutigkeit durch drei unabhängige Parameter $e_{ii} = \varepsilon_i$ bestimmt. Falls die e_i und ε_i Konstanten sind, bezeichnen wir die Transformation T mit T_1 ; sind die $e_i = e_i(\bar{t})$, $\varepsilon_i = \varepsilon_i(\bar{t})$ willkürliche (differentierbare) Funktionen der Zeit \bar{t} so bezeichnen wir T mit T_2 .

Eine Verallgemeinerung von (1), indem man $t = \bar{t} + c$, $c = \text{Konstant}$ setzt, ist für unser Problem ohne Bedeutung.

Die Menge aller T_1 bildet eine sechsgliedrige Gruppe \mathfrak{G}_1 ; die verschiedenen T_2 bilden eine unendliche Gruppe \mathfrak{G}_2 . Ausserdem haben wir zu tun mit der dreigliedrigen Drehungsgruppe, in inhomogenen Koordinaten (ξ) bestimmt durch die Gleichungen:

$$\xi_i = e_{i1} \bar{\xi}_1 + e_{i2} \bar{\xi}_2 + e_{i3} \bar{\xi}_3 \quad (i = 1, 2, 3) \quad . \quad . \quad . \quad (2)$$

Diese sei mit \mathfrak{G}_0 bezeichnet.

Deuten wir (1) als Koordinatentransformation $(x_i) \rightarrow (\bar{x}_i)$ im Euklidischen Raume, wo die (x_i) und (\bar{x}_i) auf rechtwinklige Axenkreuze O_1 bzw. O_2 zu beziehen sind, so verharren O_1 und O_2 zu einander in Ruhe, wenn eine T_1 vorliegt, während im Fall der T_2 das Axenkreuz O_2 mit wachsender Zeit jede beliebige, aus Translationen und Rotationen zusammengesetzte Bewegung, mit beliebig-variabler Geschwindigkeit relativ zu O_1 vollführen kann. Somit ist die Galilei-Newtongruppe der klassischen Mechanik als Spezialfall in \mathfrak{G}_2 enthalten.

Sei jetzt:

$$\varphi(x_1, x_2, x_3, t) = \bar{\varphi}(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{t}) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

ein Skalar. Die Funktion φ sei genügend oft differentierbar, im übrigen willkürlich. Die Betrachtung mehrerer Skalaren $\varphi_1 \dots \varphi_n$ ergibt nichts wesentlich neues und wird deshalb unterdrückt.

Wir stellen uns die folgende Aufgabe:

Es sind alle absoluten Differentialinvarianten F_1 und F_2 zu den Gruppen \mathfrak{G}_1 , bzw. \mathfrak{G}_2 gehörig, aus φ zu bilden, d.h. alle Lösungen der Gleichung:

$$\left. \begin{aligned} F\left(\varphi; \frac{\partial \varphi}{\partial x_i}; \dots \frac{\partial^n \varphi}{\partial x_{i_1} \dots \partial x_{i_n}}; \frac{\partial \varphi}{\partial t}; \frac{\partial^2 \varphi}{\partial x_i \partial t} \dots\right) = \\ = F\left(\bar{\varphi}; \frac{\partial \bar{\varphi}}{\partial \bar{x}_i}; \dots \frac{\partial^n \bar{\varphi}}{\partial \bar{x}_{i_1} \dots \partial \bar{x}_{i_n}}; \frac{\partial \bar{\varphi}}{\partial \bar{t}}; \frac{\partial^2 \bar{\varphi}}{\partial \bar{x}_i \partial \bar{t}} \dots\right) \end{aligned} \right\} \quad (4)$$

zu ermitteln, wo F gegebener Ordnung in den Ableitungen nach den (x_i) und t , und eine analytische Funktion seiner Argumente ist; und zwar kann man sich auf Polynome F beschränken. Freilich könnte man diese letzte Voraussetzung ganz fallen lassen, solange nur Differentialgleichungen für F betrachtet werden. In der Folge wird aber auch die symbolische Methode verwendet und da ist die Voraussetzung ganzer rationaler F — obgleich an sich keine wesentliche Beschränkung der Allgemeinheit — doch unentbehrlich.

Weil \mathfrak{G}_1 ein Teiler von \mathfrak{G}_2 , ist jedes F_2 auch ein F_1 ; umgekehrt wird nicht jedes F_1 auch ein F_2 sein.

§ 2. Der Reduktionssatz der F_1 .

Sei $(f_1, f_2 \dots)$ ein System ternärer Formen der ξ_i mit Koeffizienten δ :

$$f_n = \sum_{\lambda + \mu + \nu = n} \frac{n!}{\lambda! \mu! \nu!} \delta_{\lambda \mu \nu} \xi_1^\lambda \xi_2^\mu \xi_3^\nu \quad (n \geq 1) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Symbolisch ist zu schreiben:

$$f_1 = (a\xi); \quad f_2 = (b\xi)^2; \dots \quad f_n = (h\xi)^n; \dots \quad (5')$$

Die ganzen rationalen, zur Gruppe \mathfrak{G}_0 gehörigen Drehungsinvarianten der $(f_1 f_2 \dots f_n)$ bauen sich auf aus Faktoren der Gestalt:

$$\left. \begin{aligned} (a|\beta) &= a_1\beta_1 + a_2\beta_2 + a_3\beta_3 \\ (a\beta\gamma) &= \begin{vmatrix} a_1 & \beta_1 & \gamma_1 \\ a_2 & \beta_2 & \gamma_2 \\ a_3 & \beta_3 & \gamma_3 \end{vmatrix} \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

wo die a, β, γ irgendwelche Symbole $a, b, \dots h$ sind. So ist z. B. nach

(5) $a_{i_1} a_{i_2} \dots a_{i_n}$ mit $\delta_{\lambda\mu\nu}$ identisch, wenn $i = 1, 2, 3$ und die Zahlenreihe $(i_1 i_2 \dots i_n)$ λ -mal 1, μ -mal 2 und ν -mal 3 enthält ¹⁾.

Wir setzen:

$$\left. \begin{aligned} \frac{\partial^{\lambda+\mu+\nu} \varphi}{\partial x_1^\lambda \partial x_2^\mu \partial x_3^\nu} &= D_{\lambda\mu\nu} \\ \frac{\partial D_{\lambda\mu\nu}}{\partial t} &= \dot{D}_{\lambda\mu\nu}; \quad \frac{\partial \dot{D}_{\lambda\mu\nu}}{\partial t} = \ddot{D}_{\lambda\mu\nu} \text{ u.s.f.} \end{aligned} \right\} \dots \dots (7)$$

und schreiben immer transformierte Gröößen mit Strich, also $\frac{\partial^{\lambda+\mu+\nu} \bar{\varphi}}{\partial x_1^\lambda \partial x_2^\mu \partial x_3^\nu} = \bar{D}_{\lambda\mu\nu}$ u.s.f.

Für \mathfrak{G}_1 lauten die Transformationsformeln der Differentiationen:

$$\left. \begin{aligned} \frac{\partial}{\partial x_i} &= e_{i1} \frac{\partial}{\partial x_1} + e_{i2} \frac{\partial}{\partial x_2} + e_{i3} \frac{\partial}{\partial x_3} \quad (i = 1, 2, 3) \\ \frac{\partial}{\partial t} &= \frac{\partial}{\partial t} \end{aligned} \right\} \dots \dots (8)$$

d.h. der Prozesz $\frac{\partial}{\partial x_i}$ ist kogredient zu den ξ_i in (2), während $\frac{\partial}{\partial t}$ sich wie eine invariante Operation verhält. Hieraus ergibt sich die Kogredientz der Gröößen $D_{\lambda\mu\nu}$, $\dot{D}_{\lambda\mu\nu}$, $\ddot{D}_{\lambda\mu\nu}$, ... zu den Formenkoeffizienten $\delta_{\lambda\mu\nu}$ in (5), da $\varphi = \bar{\varphi}$ sich gemäsz (3) wie eine Invariante nullter Ordnung beträgt. Überdies sind die verschiedenen $D_{\lambda\mu\nu}$, $\dot{D}_{\lambda\mu\nu}$, ... algebraisch-unabhängige Gröößen. Somit gilt der:

Satz 1. Man bekommt alle F_1 p -ter Ordnung in den Ableitungen nach den (x_i) und q -ter Ordnung in den Ableitungen bzgl. t , indem man mittels (6) alle Drehungsinvarianten der Grundformen

$$f_{n,m} = \sum_{\lambda+\mu+\nu=n} \frac{n!}{\lambda! \mu! \nu!} \delta_{\lambda\mu\nu}^{(m)} \xi_1^\lambda \xi_2^\mu \xi_3^\nu \cdot \begin{pmatrix} n=1 \dots p \\ m=0, \dots, q \end{pmatrix} \dots \dots (9)$$

bildet und dann setzt:

$$\delta_{\lambda\mu\nu}^{(m)} = \frac{\partial^m D_{\lambda\mu\nu}}{\partial t^m} \dots \dots \dots (10)$$

§ 3. Die Invarianten F_2 : Allgemeines.

Während die Aufstellung aller F_1 ein leichtes war durch die Möglichkeit einer sofortigen Reduktion des Problems auf eines der algebraischen Invariantentheorie, ist dem nicht so im Falle der F_2 ; und zwar hat man für jede Differentiations-Ordnung bzgl. t mit einem neuen Problem zu tun.

¹⁾ Vgl. etwa R. WEITZENBÖCK, Invariantentheorie S. 236 (5), wo man $n = 4$ zu setzen hat. Die dortige homogenisierende Variable ist hier fortgelassen.

Den Grund für dieses Verhalten bilden die zur Gruppe \mathfrak{G}_2 gehörigen Transformationsformeln der Differentiationen:

$$\left. \begin{aligned} \frac{\partial}{\partial x_i} &= e_{i1} \frac{\partial}{\partial x_1} + e_{i2} \frac{\partial}{\partial x_2} + e_{i3} \frac{\partial}{\partial x_3} \quad (i=1, 2, 3) \\ \frac{\partial}{\partial t} &= \frac{\partial}{\partial t} - \eta_1 \frac{\partial}{\partial x_1} - \eta_2 \frac{\partial}{\partial x_2} - \eta_3 \frac{\partial}{\partial x_3} \end{aligned} \right\} \dots \quad (11)$$

wo die neuen Parameter η_i bestimmt werden durch:

$$\left. \begin{aligned} \eta_i &= e_{i1} \zeta_1 + e_{i2} \zeta_2 + e_{i3} \zeta_3 \\ \zeta_i &= \dot{e}_i + \dot{e}_{i1} \bar{x}_1 + \dot{e}_{i2} \bar{x}_2 + \dot{e}_{i3} \bar{x}_3 \end{aligned} \right\} \dots \quad (12)$$

Die Transformationsformel der $\frac{\partial}{\partial x_i}$ enthält nur Größen e_{ik} , welche von den Parametern e_i (s. o.) abhängig sind. Das gleiche gilt für die höheren Ableitungen:

$$\left. \begin{aligned} \frac{\partial^{\lambda+\mu+\nu}}{\partial x_1^\lambda \partial x_2^\mu \partial x_3^\nu} &= \left(e_{11} \frac{\partial}{\partial x_1} + e_{12} \frac{\partial}{\partial x_2} + e_{13} \frac{\partial}{\partial x_3} \right)^\lambda \cdot \\ &\quad \left(e_{21} \frac{\partial}{\partial x_1} + e_{22} \frac{\partial}{\partial x_2} + e_{23} \frac{\partial}{\partial x_3} \right)^\mu \cdot \left(e_{31} \frac{\partial}{\partial x_1} + e_{32} \frac{\partial}{\partial x_2} + e_{33} \frac{\partial}{\partial x_3} \right)^\nu \end{aligned} \right\} \quad (13)$$

Es ist also:

$$D_{\lambda\mu\nu} = \Phi(e_i, \bar{D}_{\alpha\beta\gamma}) \dots \quad (14)$$

wo Φ eine Linearform der verschiedenen $\bar{D}_{\alpha\beta\gamma}$ ist mit $\alpha + \beta + \gamma = \lambda + \mu + \nu$.

Aus (11), (14) ergibt sich:

$$\dot{D}_{\lambda\mu\nu} = \frac{\partial \Phi}{\partial t} - \eta_1 \frac{\partial \Phi}{\partial x_1} - \eta_2 \frac{\partial \Phi}{\partial x_2} - \eta_3 \frac{\partial \Phi}{\partial x_3} \dots \quad (15)$$

Diese Gleichung enthält neben den e_i die weiteren 6 Parameter \dot{e}_i und η_i :

$$\dot{D}_{\lambda\mu\nu} = \Phi_1(e_i; \dot{e}_i; \eta_i; \bar{D}_{\alpha\beta\gamma}; \dot{\bar{D}}_{\alpha\beta\gamma}) \dots \quad (15')$$

Weiterführung dieses Verfahrens ergibt:

$$\ddot{D}_{\lambda\mu\nu} = \Phi_2(e_i; \dot{e}_i; \ddot{e}_i; \eta_i; \dot{\eta}_i; \bar{D}_{\alpha\beta\gamma}; \dot{\bar{D}}_{\alpha\beta\gamma}; \ddot{\bar{D}}_{\alpha\beta\gamma}) \text{ u.s.f. } \dots \quad (16)$$

Im Allgemeinen enthält also die Transformationsgleichung der $\frac{\partial^m D_{\lambda\mu\nu}}{\partial t^m}$

die $6m+3$ unabhängigen Parameter $e_i \dots e_i^{(m)} = \frac{\partial^m e_i}{\partial t^m}$, $\eta_i \dots \eta_i^{(m-1)} = \frac{\partial^{m-1} \eta_i}{\partial t^{m-1}}$;

und ebenso viele Parameter treten in der Gleichung (4) auf, falls sie m -ter Ordnung in den Ableitungen nach t ist. Zur Aufstellung der Differentialinvarianten F_2 hat man alsdann den $6m+3$ notwendigen Bedingungen zu genügen, die das Wegfallen dieser Parameter aus (4) ausdrücken.

§ 4. Die Invarianten nullter Ordnung in t .

Wir beschränken uns jetzt auf Invarianten F_2 der Ordnungen 0 und 1 in den Ableitungen nach t ; die ersteren seien mit I , die letzteren mit K bezeichnet. Für eventuelle physikalische Anwendungen würden diese Ordnungen genügen.

Satz 2. Die Differentialinvarianten I sind mit den Invarianten nullter Ordnung in t der Gruppe \mathfrak{G}_1 identisch.

Beweis. Der Satz ist ohne weiteres klar, denn die Transformationsformeln der Derivierten nach den (x_i) sind für beide Gruppen identisch.

Die einfachsten Vertreter der Invarianten I sind, symbolisch und nicht-symbolisch geschrieben:

$$\left. \begin{aligned} I_1 &= (a|a) = \sum_{i=1}^3 \left(\frac{\partial \varphi}{\partial x_i} \right)^2; \\ I_2 &= (b|b) = \sum_i \frac{\partial^2 \varphi}{\partial x_i^2}; \\ I_3 &= (a|b)^2 = \sum_{i,k} \frac{\partial \varphi}{\partial x_i} \cdot \frac{\partial \varphi}{\partial x_k} \cdot \frac{\partial^2 \varphi}{\partial x_i \partial x_k}; \end{aligned} \right\} \dots \dots (17)$$

u. s. f.

§ 5. Die Invarianten erster Ordnung in den Ableitungen nach t .

Die Gleichung (4) bekommt jetzt die Gestalt:

$$K(D_{\lambda_p \mu_p \nu_p}; D_{\alpha_q \beta_q \gamma_q}) = K(\bar{D}_{\lambda_p \mu_p \nu_p}; \bar{D}_{\alpha_q \beta_q \gamma_q}) \quad \left(\begin{matrix} p=1 \dots m \\ q=1 \dots m' \end{matrix} \right). \quad (18)$$

Schreiben wir dies in der Form:

$$K = \bar{K} \dots \dots \dots (18')$$

so enthält das zweite Glied keine Transformationsparameter, während durch den Übergang $(x_i; t) \rightarrow (\bar{x}_i; \bar{t})$ im ersten Glied die neun Gröößen $(\varepsilon_i, \bar{\varepsilon}_i, \eta_i)$ auftreten, die sich wegheben müssen.

Man erhält so 6 Bedingungen, n.l. $\frac{\partial K}{\partial \eta_i} = 0, \frac{\partial K}{\partial \varepsilon_i} = 0$ in der Form homogener linearer partieller Differentialgleichungen mit konstanten Koeffizienten. Ihre Integration ergibt K in einer Gestalt, die noch 3 Parameter ε_i enthält und die eine Reduktion der K auf Drehungsinvarianten ermöglicht. An die Stelle der Formkoeffizienten, die jene Drehungsinvarianten bilden, treten hier 1^o. die verschiedenen Gröößen $D_{i\mu\nu}$, 2^o. gewisse Determinanten 7^{ten} Grades aus den verschiedenen $D_{\lambda\mu\nu}$ und $\bar{D}_{\lambda\mu\nu}$ gebildet.

Die Indizestripel $(\lambda_1 \mu_1 \nu_1)$, $(\lambda_2 \mu_2 \nu_2)$, $(\lambda_3 \mu_3 \nu_3)$ sind voneinander verschieden, es gibt also in ∇_{123} eine Grösze $D_{h_1 h_2 h_3}$, die sich von allen übrigen in ∇_{123} auftretenden Gröszen $D_{k_1 k_2 k_3}$ unterscheidet durch die Existenz einer der Beziehungen:

$$1^0. \quad h_1 > k_1.$$

$$2^0. \quad h_1 = k_1; \quad h_2 > k_2.$$

$$3^0. \quad h_1 = k_1; \quad h_2 = k_2; \quad h_3 > k_3.$$

Es werde $D_{h_1 h_2 h_3}$ als „Maximalelement“ der Determinante ∇_{123} angedeutet. ∇_{123} kann nicht identisch verschwinden solange der Minor von $D_{h_1 h_2 h_3}$ nicht identisch verschwindet. Dieser Minor aber besitzt auch ein „Maximalelement“, mit von 0 verschiedenem Variabelkoeffizient, kann also tatsächlich nicht identisch verschwinden.

Der Forderung $\frac{\partial K}{\partial \dot{D}_{\lambda_1 \mu_1 \nu_1}} \neq 0 \dots \frac{\partial K}{\partial \dot{D}_{\lambda_m \mu_m \nu_m}} \neq 0$ kann man nur genügen für $m \geq 4$, es gibt also nur für solche Werte m Differentialinvarianten K .

Jetzt gibt der Hilfssatz 1 zusammen mit (25), (26) für das allgemeine Integral von (27):

$$K = \Phi(\Delta_4 \dots \Delta_m; D_{\alpha_1 \beta_1 \gamma_1} \dots D_{\alpha_m \beta_m \gamma_m}) \quad \dots \quad (29)$$

§ 8. Das Verschwinden der $\frac{\partial K}{\partial \varepsilon_i}$.

Aus (29) folgert man:

$$\frac{\partial K}{\partial \varepsilon_i} = \sum_{p=4}^m \frac{\partial \Phi}{\partial \Delta_p} \cdot \frac{\partial \Delta_p}{\partial \varepsilon_i} = 0 \quad (i = 1, 2, 3) \quad \dots \quad (30)$$

Wir schreiben zunächst Δ_p in einer Form, die keine Gröszen η_i mehr enthält:

$$(25): \quad \Delta_{i_1 \dots i_4} = \frac{\partial (D_{i_1} D_{i_2} D_{i_3} D_{i_4})}{\partial (t x_1 x_2 x_3)} \quad \dots \quad (25')$$

also:

$$\left. \begin{aligned} \frac{\partial \Delta_p}{\partial \varepsilon_i} = & \nabla_{23p} \frac{\partial}{\partial \varepsilon_i} \left(\frac{\partial D_1}{\partial t} \right) - \nabla_{13p} \frac{\partial}{\partial \varepsilon_i} \left(\frac{\partial D_2}{\partial t} \right) + \\ & + \nabla_{12p} \frac{\partial}{\partial \varepsilon_i} \left(\frac{\partial D_3}{\partial t} \right) - \nabla_{123} \frac{\partial}{\partial \varepsilon_i} \left(\frac{\partial D_p}{\partial t} \right) \end{aligned} \right\} \quad (31)$$

Weiter ist:

$$\frac{\partial D}{\partial t} = \frac{\partial D}{\partial t_{(i)}} + \sum_{i=1}^3 \frac{\partial D}{\partial \varepsilon_i} \varepsilon_i \quad \dots \quad (32)$$

also:

$$\frac{\partial}{\partial \varepsilon_i} \left(\frac{\partial D}{\partial t} \right) = \frac{\partial D}{\partial \varepsilon_i} \quad \dots \quad (32')$$

$$(31): \quad \frac{\partial \Delta_p}{\partial \varepsilon_i} = \nabla_{23p} \frac{\partial D_1}{\partial \varepsilon_i} - \nabla_{13p} \frac{\partial D_2}{\partial \varepsilon_i} + \nabla_{12p} \frac{\partial D_3}{\partial \varepsilon_i} - \nabla_{123} \frac{\partial D_p}{\partial \varepsilon_i} \quad (31')$$

Zur Berechnung von $\frac{\partial D_{\lambda\mu\nu}}{\partial \varepsilon_j}$ schreiben wir:

$$\frac{\partial D_{\lambda\mu\nu}}{\partial \varepsilon_j} = \sum_{(ik)} \frac{\partial D_{\lambda\mu\nu}}{\partial e_{ik}} \cdot \frac{\partial e_{ik}}{\partial \varepsilon_j} \quad (j = 1, 2, 3) \quad (33)$$

wo im Ausdruck $\frac{\partial D}{\partial e_{ik}}$ den Abhängigkeitsbeziehungen der e_{ik} keine Rechnung getragen wird. Die Formel (13) gibt sodann:

$$\left. \begin{aligned} \frac{\partial D_{\lambda\mu\nu}}{\partial e_{1k}} &= \lambda \frac{\partial D_{\lambda-1, \mu, \nu}}{\partial x_k}; & \frac{\partial D_{\lambda\mu\nu}}{\partial e_{2k}} &= \mu \frac{\partial D_{\lambda, \mu-1, \nu}}{\partial x_k}; & \frac{\partial D_{\lambda\mu\nu}}{\partial e_{3k}} &= \nu \frac{\partial D_{\lambda, \mu, \nu-1}}{\partial x_k} \end{aligned} \right\} \quad (k = 1, 2, 3) \quad (34)$$

Die Größen $\frac{\partial e_{ik}}{\partial \varepsilon_j}$ berechnet man leicht durch Differentiation der Orthogonalitätsbedingungen der e_{ik} , indem man beachtet dass:

$$\frac{\partial e_{il}}{\partial \varepsilon_j} = \delta_{lj} = \begin{cases} 0 & \text{für } i \neq j \\ 1 & \text{,, } i = j \end{cases} \quad (35)$$

So findet man aus (33), (34):

$$\left. \begin{aligned} \varepsilon \frac{\partial D_{\lambda\mu\nu}}{\partial \varepsilon_1} &= e_{12} e_{13} \Gamma_1 + e_{12} e_{23} \Gamma_2 + e_{13} e_{32} \Gamma_3 \\ \varepsilon \frac{\partial D_{\lambda\mu\nu}}{\partial \varepsilon_2} &= e_{21} e_{13} \Gamma_1 + e_{23} e_{21} \Gamma_2 + e_{23} e_{31} \Gamma_3 \\ \varepsilon \frac{\partial D_{\lambda\mu\nu}}{\partial \varepsilon_3} &= e_{31} e_{12} \Gamma_1 + e_{32} e_{21} \Gamma_2 + e_{31} e_{32} \Gamma_3 \end{aligned} \right\} \quad (36)$$

wo:

$$\varepsilon = e_{32}^2 - e_{23}^2 = e_{13}^2 - e_{31}^2 = e_{21}^2 - e_{12}^2 \quad (37)$$

und:

$$\left. \begin{aligned} \Gamma_1(D_{\lambda\mu\nu}) &= \mu D_{\lambda, \mu-1, \nu+1} - \nu D_{\lambda, \mu+1, \nu-1} \\ \Gamma_2(D_{\lambda\mu\nu}) &= \nu D_{\lambda+1, \mu, \nu-1} - \lambda D_{\lambda-1, \mu, \nu+1} \\ \Gamma_3(D_{\lambda\mu\nu}) &= \lambda D_{\lambda-1, \mu+1, \nu} - \mu D_{\lambda+1, \mu-1, \nu} \end{aligned} \right\} \quad (38)$$

Wegen (31'), (36) und:

$$\begin{vmatrix} e_{12} & e_{13} & e_{12} & e_{23} & e_{13} & e_{32} \\ e_{21} & e_{13} & e_{23} & e_{21} & e_{23} & e_{31} \\ e_{31} & e_{12} & e_{32} & e_{21} & e_{31} & e_{32} \end{vmatrix} = \varepsilon^2 \neq 0 \quad (39)$$

folgt nun aus (30):

$$\sum_{p=4}^m \frac{\partial \Phi}{\partial \Delta_p} C_i(p) = 0. \quad (i = 1, 2, 3) \quad (40)$$

wo:

$$C_i(p) = \nabla_{23p} \Gamma_i(1) - \nabla_{13p} \Gamma_i(2) + \nabla_{12p} \Gamma_i(3) - \nabla_{123} \Gamma_i(p) \quad (41)$$

wenn wir zur Abkürzung schreiben:

$$\Gamma_i(\tau) = \Gamma_i(D_\tau) = \Gamma_i(D_{\lambda_\tau \mu_\tau \nu_\tau}) \quad (i = 1, 2, 3) \quad . \quad . \quad . \quad (42)$$

Die Gleichungen (40) stellen wieder ein System dreier linearer partieller Differentialgleichungen mit konstanten Koeffizienten dar, denn die $C_i(p)$ enthalten im Gegensatz zu den Δ_p nur Variablen $D_{\lambda \mu \nu}$, sind also als von den Δ_p unabhängige Größen zu betrachten.

Auch hier gibt es, wie bei den Gleichungen (27) eine zu diskutierende Koeffizientenmatrix $\|C_i(p)\|$, die aber im Gegensatz zur Matrix (28) für besondere Parameterwerte $(\lambda_1 \mu_1 \nu_1 \dots \lambda_m \mu_m \nu_m)$ und für $m \geq 6$ vom Range 2 sein kann.

Dennoch erscheint es zweckmässiger die Erörterung der Fälle $m = 4, 5, 6$ und die Untersuchung der Matrix $\|C_i(p)\|$ für $m \geq 7$ auf später zu verschieben, nachdem der allgemeine Fall erledigt ist. Das Ergebnis führt nämlich dahin, dass in den Ausnahmefällen tatsächlich Differentialinvarianten K existieren, dass diese sich aber ganz den allgemeinen Formeln fügen. Doch ist ein solches Verhalten nicht von vornherein zu erwarten.

[illegible]

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